Duration-Enhancing Overlay Strategies for Defined-Benefit Pension Plans

John M. Mulvey
Professor
Bendheim Center for Finance
Operations Research and Financial Engineering Department
Princeton University
mulvey@princeton.edu

Woo Chang Kim
Ph.D. Candidate
Operations Research and Financial Engineering Department
Princeton University
wookim@princeton.edu

Yi Ma
Ph.D. Candidate
Operations Research and Financial Engineering Department
Princeton University
yma@princeton.edu

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Abstract

Many large corporate and public pension trusts remain underfunded since the 2001-2002 recessionary periods. These plans are challenged by global demographic trends and the recent slowing economic conditions. We show that a special overlay strategy can improve performance and reduce risks by adding duration to the portfolio. The approach combines elements of liability-driven investing and asset-liability management. Versions of the strategy are evaluated via historical data. In addition, the strategy is tested with a widely employed, forward-looking economic projection system.

1. Introduction

In many countries, there is a concern about demographic trends associated with providing adequate pensions to retired workers. For example, the workforce in Japan is expected to shrink to from 84 million to 74 million, while the number of retirees will increase from 26 million to 36 million by 2030. In Europe, the population aged 65 and older as a ratio of the working age population will increase from 25% in 2008 to 53% in 2060. Given these headwinds, companies who sponsor pension trusts will benefit from improving their investment performance.

Subsequently, there has been a trend to modify defined benefit (DB) pension plans. First, pension plans have lost their surpluses since the 2001-2002 drop in equity markets and the commensurate decrease in interest rates. Second, changes in regulation in the U.S. have increased the penalty for pension deficits. Third, the bankruptcy of a number of large firms over the past few years has shown that traditional pension valuations may be too optimistic when a firm experiences financial distress.

In this paper, we show that a specialized version of overlay strategies can help DB pensions to achieve their financial goals more efficiently. In addition to empirical tests using historical data, we adopt a forward looking model under the asset-liability management (ALM) framework[1] that integrates assets and liabilities in stochastic simulations.

2. Two approaches in core portfolio management for DB pension plans

In defined pension plans, liabilities can be understood as a form of a fixed income security. Retirees are guaranteed to receive a pre-determined amount of money in the future from the pension plan. Therefore, a major source of risk involves the discount rate due to changes in interest rates. Thus, it is natural to construct the core portfolios (traditional assets such as equities and bonds) to hedge interest rate risks by matching duration of assets with liabilities. Due to the nature of pensions, duration on liabilities is rather long. For instance, a typical pension plan possesses roughly 12.6 years duration[2]. Therefore, in order to match or immunize these liabilities, the core portfolios are mainly composed of long term fixed income securities such as long dated zero-coupon bonds. As mentioned, this type of conservative pension plan management scheme prioritizes the duration matching (immunization) to replicate the future cash outflows, so the portfolios have usually higher weights on long-term bonds.

However, there is a downside to these conservative approaches. In particular, long-term bonds have lower expected returns than equities; therefore, these strategies will give rise to expected contributions

[1] See Consigli and Dempster (1998), Mulvey and Ziemba (1998), Muralidhar, et al. (1999), Ziemba (2003), Fabozzi, et al. (2004) and Zenios and Ziemba (2006).
[2] Towers-Perrin liability index has a duration of 12.6 years.

from the sponsoring company. Historically, fixed income securities have underperformed compared to other investment vehicles such as stocks, so the portfolios mainly formed with bonds would have lower returns. Thus, they are unlikely to provide high growth rate in the core portfolios enough to cover the growth in liabilities, which will force high contributions. With the changes in demographic trends of lower birth rates and extended life expectancies in most of developed countries, poor performance in core assets will burden sponsoring companies more as time goes on. Further, there could be macro impacts to the economy if a large number of pension plans move their asset allocation to fixed-income categories. Considering the size of pension plans, for example, their concentration on the bond market will decrease returns.

In contrast to the conservative matching approaches, another approach puts more emphasis on the performance of the core portfolios. Under this framework, the core portfolios are mainly constructed with investment vehicles with relatively high performance such as stocks, private equities, hedge funds, etc. Due to higher expected performance, it could ease the burden of large future contributions by the sponsoring companies. However, since the core assets are less correlated to the liabilities as compared to the conservative approaches due to the duration mismatch, future contributions become more volatile. In other words, although the performance-oriented approach might reduce the contributions on average, the worst case over shorter time periods will be more significant to the sponsoring companies than that of the duration matching approach. The main source of risk arises from the shorter duration on core assets, since duration of non-fixed income securities are relatively short, if not zero.

3. Duration enhancing overlay strategies

For large DB pensions, the traditional approach is to hire an outside manager, such as commodity trading advisor. The manager can develop and implement an overlay strategy. Herein, traditional assets in the core portfolio are employed as margin capital for targeted positions in the futures/forward markets. Overlay strategies have been popular with institutional investors in several areas including managing currencies. Mulvey, et al. (2006) and Mulvey, et al. (2007) show the benefits of general overlay strategies within asset allocation and ALM. These strategies seek to widen diversification of the portfolio, while generating higher growth rates. Popular strategies employ three general domains: commodities, fixed-income, and currencies. In each case, trading volume in the futures market is large enough so that investors can quickly rebalance the portfolio mix as conditions warrant.

Importantly, these types of overlay strategies must be implemented in conjunction with careful risk management. There are examples of organizations that experienced severe difficulties since they were either unaware of possible risks in their positions, or did not take adequate action to protect their capital.

A prime example occurred in 1994 involving Orange County, California. The treasurer Edward Citron wished to increase returns for the county. He managed to take on a substantial bet on long-dated interest rates through swaps and related investments, and a short bet on short term interest rates. The drop in his position in 1994 due to a structural shift in the yield curve was not foreseen. Subsequently, Orange County declared bankruptcy.

In this paper, we focus on a specialized type of overlay, called duration-enhancing overlay (DEO) in order to improve the performance of defined-benefit pension plans. The basic idea is straightforward: it is a zero investment strategy constructed by taking long future positions in long-dated treasury bonds, while taking opposite positions in shorter term treasury securities.

Here, we evaluate the fixed-percentage DEO strategy. It provides a special case of the longstanding fixed-mix rule. The investor modifies her portfolio over time in order to enforce a constant percentage target. For example, an investor might select 75% as a target for DEO. This investor will reset the portfolio values on a recurring basis. Fixed-mix rules have been shown to be optimal for long term investors. For example, see the early work by Samuelson (1969) and Merton (1969); also see Fernholtz (1982), Luenberger (1998), and Mulvey, et al. (2005).

The logic for employing DEO for DB pension plans is intuitive. Suppose the core portfolio is constructed from various investment vehicles and DEO will be employed in addition to the traditional assets. Then, the DEO strategy works similar to other levered investments, since long-term bonds generally will have higher returns than cash. So in traditional portfolio management contexts, the asset values can be written as in the following equation.

$$W_{t+1} = W_t[1 + \sum x_{i,t} r_{i,t} + x_{DEO,t} r_{DEO,t}],$$

where W_t: wealth at t,

$x_{i,t}$: weight on asset i at t,

$r_{i,t}$: return on asset i at t,

$x_{OL,t}$: weight on DEO at t,

$r_{DEO,t}$: return on DEO at t,

Since DEO is implemented independently to the core assets, the sum of the weights on core assets ($\sum x_{i,t}$) is 1. Under the performance-oriented approach, the effects of employing DEO depend on the trade-off between the diversification benefits from the low correlation between DEO and the core portfolio and increased risks due to the position on DEO, if the asset-only aspect is considered.

However, when the pension plan is considered as an integrated system of asset and liability, DEO can provide benefits beyond enhanced asset performance. The answer lies with increased duration. Assuming linearity, the duration of core assets with DEO is:

$$D_{Asset,t} = \sum x_{i,t} D_{i,t} + x_{DEO,t} D_{DEO,t},$$

where $D_{Asset,t}$: duration of core assets at t,

$x_{i,t}$: weight on asset i at t,

$D_{i,t}$: duration on asset i at t,

$x_{OL,t}$: weight on DEO at t,

$D_{DEO,t}$: duration on DEO at t.

In general,

$$D_{DEO,t} \approx D_{LongBond,t} - D_{Cash,t} > 0.$$

Since DEO adds duration to the asset portfolio, the performance pattern of core assets would become more similar to that of liabilities as the interest rate changes over time. This gives investors more room to invest in high performance securities with low duration while controlling exposure to the interest risk at a reasonable level. In a long run, the approach improves investment performance for long-term investors, causing healthier plans and lower contributions.

4. Empirical Results with Historical Data

This section describes a set of empirical results of applying the specialized overlay strategy over the 26-year period (1982 to 2007). In particular, we evaluate the duration-based strategy presented in the previous section with a focus on potentials to extend the duration of core assets. To keep the discussion relatively simple, we focus on asset allocation decisions only and ignore contribution and payout decisions. The next section will take up the full pension plan environment.

To analyze the impact of the DEO strategies, we apply two standard fixed-mix rules: 60-40 (60% equities and 40% bonds) and 70-30 (70% equities and 30% bonds). We employ the S&P 500 index and the USA 10-year government bond index as proxies for equities and bonds, respectably. Under the fixed-mix approach, the investor must rebalance her portfolio at the beginning of each period (monthly in our empirical tests) to the target percentages (60-40 or 70-30). Thus, stocks must be sold (bought) if they

outperform (underperform) bonds over the previous time period. The Towers-Perrin liability index[3] is employed as a proxy on liabilities to replicate the long duration of the liability properly[4]. In addition, as mentioned, DEO is modeled by taking long position on 10-year bonds and short position on 3month t-bills. Although DEO should be constructed within the futures market, we adopt spot returns on 10-year bonds and t-bills to keep the analysis relatively simple. See Table 1 for the performance of the six main ingredients.

Panel 1: Asset-Only Performance Measures

	Liability	Equity	Bond	Tbill	60-40	70-30	DEO
Annual Geometric Mean Return	8.15%	13.06%	10.24%	5.38%	12.26%	12.50%	4.63%
Volatility	10.39%	14.66%	8.38%	0.70%	9.88%	10.93%	8.33%
Maximal Drawdown	24.35%	44.73%	11.64%	0.00%	20.80%	27.45%	19.06%
Sharpe Ratio	0.267	0.524	0.584	N/A	0.699	0.653	-0.090
Excess return/Max. Drawdown	0.114	0.172	0.418	N/A	0.331	0.259	-0.039

Panel 2: Short-term risk measures

	Liability	Equity	Bond	Tbill	60-40	70-30	DEO
Mean Return for 5% Worst Month	-5.81%	-8.98%	-4.09%	0.08%	-5.12%	-6.03%	-4.55%
Mean Return for 1% Worst Month	-8.24%	-14.00%	-5.33%	0.08%	-7.56%	-9.00%	-5.77%
Return for Worst Month	-10.00%	-21.53%	-6.71%	0.07%	-10.45%	-13.22%	-6.79%

Panel 3: Relative performance measures

	Liability	Equity	Bond	Tbill	60-40	70-30	DEO
Correlation to Liability	1.000	0.284	0.840	0.043	0.537	0.459	0.841
Tracking Error	0.00%	4.90%	2.09%	-2.77%	4.11%	4.35%	-3.52%
Std(Tracking Error)	0.00%	15.37%	5.65%	10.38%	9.76%	11.10%	5.63%
Information Ratio	N/A	0.319	0.370	-0.267	0.421	0.392	-0.625

Table 1: Performance of the key strategies

The performance of liability, equity, bonds, 60/40, T-bills, and DEO over the time period Jan 1982 to Dec 2007 is listed. The historical performance of the two benchmark strategies (70-30 and 60-40) is good (Sharpe ratios above .6), while their correlations to the liabilities are relatively low. On the other hand, the standalone performance of DEO has been moderate (Sharpe ratio: -0.09, Information ratio: -0.625) and the level of risk exposure is relatively high, but the returns from DEO are highly correlated to the liabilities. Also equities are the best performing asset (13% per annum), yet one of the least correlated assets to liability (0.284).

[3] For the period of 1982 to 2000, the liability growth rates are extrapolated based on the Towers-Perrin liability construction rules. Growth rates for 2001 to 2007 are directed obtained from the Towers-Perrin liability index.

[4] We also conduct the same tests employing the 30-year government bond index as a different proxy for the liability. The tests yield similar results as the tests with Towers-Perrin liability index. See Appendix A.

Several points should be addressed in Table 1. First, although the historical performance of the two benchmark strategies (70-30 and 60-40) are good (Sharpe ratios above .6), they have relatively low correlations to the liabilities (0.537, and 0.459, respectably). When duration of core assets and liabilities are well matched, changes in their wealth due to a parallel shift in the yield curve are similar. Thus, as the duration mismatch declines, asset returns and liability growth rates become more correlated. In this sense, the correlation between monthly returns of liabilities and strategies can be employed as an indirect measure for a level of duration matching. Therefore, low correlations between returns of benchmark strategies and liabilities imply that the conventional approaches have suffered from the short core portfolio duration. All of the investment vehicles employed in this section generally possess lower duration than liabilities. The main issue is to lengthen the core asset duration.

Next, as a standalone investment vehicle, the performance of DEO has been rather poor (Sharpe ratio: -0.09, Information ratio: -0.625), while the level of risk exposure is relatively high. Especially, short term risk measures possess similar values to those of benchmark strategies. Nevertheless, the returns from DEO are highly correlated to the liabilities, implying that it could potentially be a useful tool to modify the return pattern and duration of the core portfolio. An important observation is that equities are the best performing asset (13% per annum), yet one of the least correlated assets to liability (0.284). Therefore, the key to the successful DB pension plan management is to add duration while properly utilizing high performance assets such as equities.

Table 2 depicts the summary statistics for benchmark strategies (panel 1: 60-40, panel 2: 70-30) after applying several levels of DEO (0 to 200%). Note that weights on DEO are relative to the wealth on core assets. For instance, when 100% DEO is employed, the investor should take a long position on the long-term bond worth the current core assets' wealth, and a short position on T-bill worth the same amount. The portfolios of benchmark strategies and DEO are rebalanced at the beginning of every month to ensure the desired level of the DEO at every juncture.

Panel 1: 60-40

1. Asset Performance Measures

DEO (%)	0	25	50	75	100	125	150	175	200
Annual Return (%)	12.2	13.5	14.7	15.8	16.9	18.0	19.1	20.1	21.0
Volatility (%)	9.8	11.0	12.4	13.9	15.6	17.4	19.2	21.1	23.0
Maximal Drawdown (%)	20.8	16.8	16.5	20.6	25.0	29.1	33.1	36.9	40.6
Sharpe Ratio	0.69	0.73	0.75	0.75	0.74	0.73	0.71	0.69	0.68
Excess return/Max. Drawdown	0.33	0.48	0.56	0.50	0.46	0.43	0.41	0.39	0.38

2. Short-term risk measures

DEO (%)	0	25	50	75	100	125	150	175	200
Return for 5% Worst Month (%)	-5.1	-5.4	-6.1	-6.9	-7.8	-8.9	-10.0	-11.1	-12.2
Return for 1% Worst Month (%)	-7.5	-7.6	-8.1	-9.0	-10.0	-11.2	-12.5	-13.8	-15.1
Return for Worst Month (%)	-10.4	-9.0	-8.8	-10.4	-12.1	-13.7	-15.3	-16.9	-18.6

3. Relative Performance Measures

DEO (%)	0	25	50	75	100	125	150	175	200
Correlation to Liability	0.53	0.64	0.71	0.75	0.78	0.80	0.82	0.83	0.83
Tracking Error (%)	4.1	5.3	6.5	7.7	8.8	9.9	10.9	11.9	12.9
Std(Tracking Error) (%)	9.7	9.0	8.8	9.1	9.8	10.9	12.2	13.7	15.3
Information Ratio	0.42	0.58	0.73	0.84	0.89	0.91	0.89	0.87	0.84

Panel 2: 70-30

1. Asset Performance Measures

DEO (%)	0	25	50	75	100	125	150	175	200
Annual Return (%)	12.5	13.7	14.9	16.1	17.3	18.4	19.4	20.4	21.4
Volatility (%)	10.9	11.8	13.0	14.4	16.0	17.6	19.4	21.2	23.0
Maximal Drawdown (%)	27.4	23.4	19.6	20.1	24.4	28.6	32.6	36.4	40.1
Sharpe Ratio	0.65	0.70	0.73	0.74	0.74	0.73	0.72	0.71	0.69
Excess return/Max. Drawdown	0.25	0.35	0.48	0.53	0.48	0.45	0.43	0.41	0.40

2. Short-term risk measures

DEO (%)	0	25	50	75	100	125	150	175	200
Return for 5% Worst Month (%)	-6.0	-6.1	-6.5	-7.1	-8.0	-8.9	-9.9	-11.0	-12.1
Return for 1% Worst Month (%)	-9.0	-8.8	-9.1	-9.5	-10.4	-11.4	-12.5	-13.8	-15.0
Return for Worst Month (%)	-13.2	-11.7	-10.3	-10.4	-12.0	-13.7	-15.3	-16.9	-18.5

3. Relative Performance Measures

DEO (%)	0	25	50	75	100	125	150	175	200
Correlation to Liability	0.45	0.57	0.65	0.71	0.75	0.78	0.80	0.81	0.82
Tracking Error (%)	4.3	5.6	6.8	8.0	9.1	10.2	11.3	12.3	13.3
Std(Tracking Error) (%)	11.1	10.3	10.0	10.1	10.6	11.5	12.7	14.1	15.6
Information Ratio	0.39	0.53	0.67	0.78	0.85	0.88	0.88	0.87	0.85

Table 2: Performance of key strategies after applying DEO

The summary statistics for benchmark strategies (panel 1: 60-40, panel 2: 70-30) after applying several levels of DEO (0 to 200%) are illustrated. Applying DEO improves the correlation significantly, implying that it is a useful tool to increase the duration on core assets. However, it becomes less efficient after a certain level (100%). As the DEO is employed more than 100% level, the short-term risks increase significantly, while the performance enhancement is less so.

Indeed, adding the DEO increases correlations of core asset portfolios to the liability significantly. For instance, applying 100% DEO improves the correlation from 0.54 to 0.79 for 60-40 and from 0.46 to 0.75 for 70-30. As the level of the DEO strategies increase, the returns and the information ratios get higher, in addition to the increment of correlations.

The test results also indicate that investors should be careful applying DEO above a certain level. For instance, for 70-30 strategy, the expected shortfall at 5% level (-mean return for 5% worst month) increases only by 2% for the first 100% of DEO, but it is 4% for the next 100%. Also, the maximal drawdown gets even better when the DEO is employed up to 100%, while it worsens significantly (24.5% → 40%) for the next 100%. Note that the relative performance measures such as correlations to liability should be prioritized to asset-only measures, for the main issue in the DB pension plan management is to match the future out-stream of the cash flows of the liability with the core assets. However, short term risk measures should be carefully considered along with the relative measures to achieve investment goals. See Figure 1 for the trend of performance changes due to different levels of the DEO strategies.

The implications are clear. The DEO strategies can help pension plans to reduce their exposure to interest rate risk factors. However, they should be implemented only at a modest level (roughly up to 100%) in order to avoid another Orange County incident.

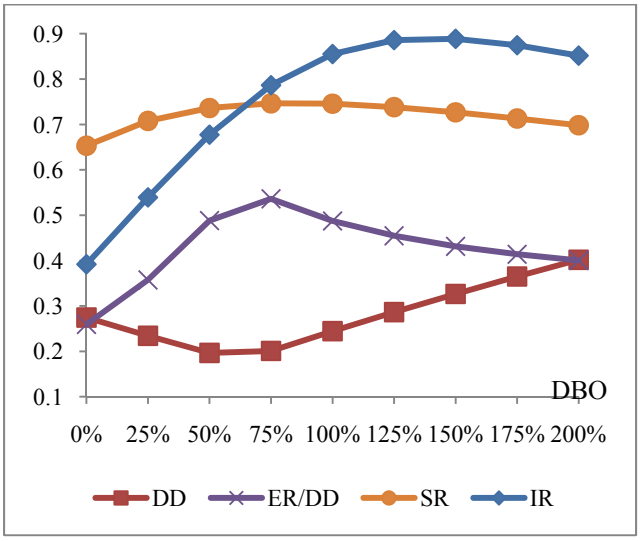

Figure 1: Comparisons on portfolio performance at different levels of DEO
(DD: maximal drawdown, ER: excess return, SR: Sharpe ratio, IR: information ratio)
Several performance measures with different DEO levels are depicted. It illustrates that the DEO is an efficient investment tool only up to a certain level (100%). When it is employed excessively, it increases risks more than performance.

5. Empirical results with a forward-looking scenario generator (CAP:Link)

Next, we evaluate the DEO strategies under the asset-liability management context. ALM models have been implemented in numerous pension and insurance studies (Dert (1995), Peskin (1997), Boender, et al. (1998), Ryan and Fabozzi (2003), Hilli et al. (2003), Olson (2005), and Mulvey, et al. (2008)). In these models, there are three basic ingredients: a system for generating future scenarios, a set of rules for significant decisions at each time period, and an algorithm for selecting the best set of decisions.

We employ the CAP:Link scenario generator for the empirical tests. This system has been employed widely throughout the world over the past 15 years (Mulvey, Gould, and Morgan (2000)). The generator employs a cascade structure, in which a set of economic factors are modeled at the highest level (interest rates, GDP, etc.) with stochastic equations. Returns of assets are modeled as functions of the economic factors at each time period for each scenario (Figure 3.2). This approach guarantees a consistent relationship between interest rates needed for discounting liabilities and asset returns including bond categories.

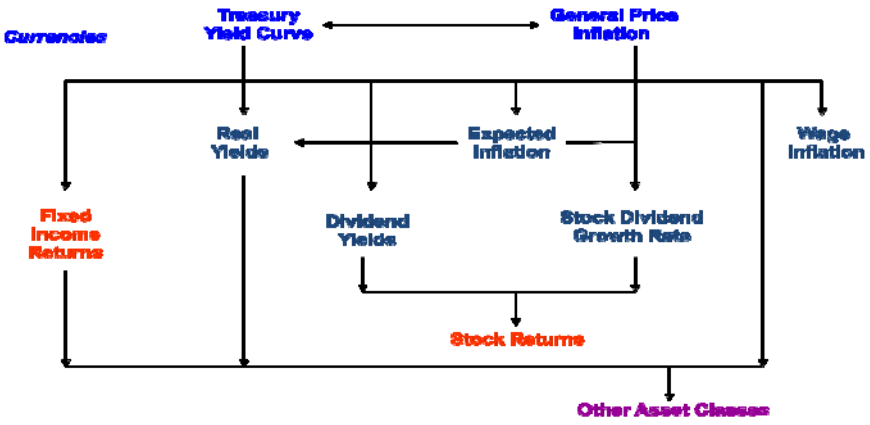

Figure 3.2: CAP:Link – cascade of stochastic processes
The CAP:LINK system models the returns of assets as functions of economic factors at each time period under each scenario. Its cascade structure can guarantee a consistent relationship between interest rates and asset returns.

We assume that the investment mix is rebalanced to a target asset mix at the beginning of each period. The fixed mix rule has become a norm in the DB pension plan management domain. For instance, it is now common for large pension plans such as TIAA-CREF to rebalance portfolios back to the chosen weights at the client's requests. A simple and intuitive explanation on the benefits of the fixed mix rules can be found in Luenberger (1997).

Let's assume that there are n stocks whose mean return is $r \in \mathbb{R}^n$ and covariance matrix $\Sigma \in \mathbb{R}^{n \times n}$. With a normality assumption, the return of the fixed mix portfolio with weight *w* follows

$$N\left(w^T r + \frac{1}{2}\sum_{i=1}^{n} w_i \sigma_i^2 - \frac{1}{2}\sigma_p^2,\ \sigma_p^2\right) \equiv N\left(w^T r + \frac{1}{2}\sum_{i=1}^{n} w_i \sigma_i^2 - \frac{1}{2} w^T \Sigma w, w^T \Sigma w\right) .$$

The extra term in the expected return compared to the Markowitz model, $(\sum_{i=1}^{n} w_i \sigma_i^2 - \sigma_p^2)/2$, which is often called rebalancing gains or volatility pumping, illustrates the benefits of the fixed mix rule. For simplicity, suppose all of stocks have the same expected return (*r*) and volatility (σ), and the correlation is ρ for any given pair. Then, the rebalancing gain for an equal-weighted portfolio becomes

$$\frac{1}{2}\left\{\sum_{i=1}^{n} \frac{1}{n}\sigma^2 - \left(\frac{1}{n} \quad \cdots \quad \frac{1}{n}\right)\Sigma\left(\frac{1}{n} \quad \cdots \quad \frac{1}{n}\right)^T\right\} = \frac{(n-1)\sigma^2(1-\rho)}{2n} ,$$

whose value is always positive, unless all stocks are perfectly correlated.

The best decision variables are selected via multi-period optimization techniques. Given the set of scenarios generated from CAP:LINK and the fixed mix rule, we optimize the multi-period portfolio models taking the initial investments and the rebalancing decisions at the beginning of each time period into account. In general, these models cannot be formulated as a convex program[5], so we solve each

[5] Two issues arise upon formulating the model into a convex problem. First, since we employ the fixed-mix rule under multi-period setting (n=5, 10), the final wealth becomes a polynomial of order n, which is a non-convex function. Second, most of the problems include constraints related to funding ratio. Since it is defined as the ratio of wealth to outstanding liability, the problems become non-convex.

problem several thousand times with different initial solutions[6]. This procedure is repeated with several risk aversion levels to generate efficient frontiers.

5.1. Data Summary and Problem Settings

The number of the generated scenarios (paths) is equal to 500 for the empirical tests. In each scenario, annual returns for ten core assets with one DEO strategy are generated for the 10-year sample period. Liabilities and benefit payments are also generated accordingly. Table 3 summarizes the data generated by CAP:LINK system.

Panel 1: Core assets and DEO strategies

Core Asset Classes (10 Assets)	Return (%)	Volatility (%)	Drawdown (%)	Duration	Correlation to Liability
LB Agg. Bond Index	5.66	2.97	3.31	4.59	0.80
Long HQ Bond	5.85	5.96	21.34	11.53	0.95
10-Yr Tips	5.20	3.15	5.38	6.68	0.54
Large-Cap Stock Index	8.20	12.81	28.61	0.00	0.40
Cash Equivalent	4.33	3.80	0.00	0.25	-0.08
Hi-Yield Bond Index	6.45	11.69	17.47	6.03	0.45
Real Estate Index	7.03	18.10	21.83	0.00	0.02
Mid-Small Stock Index	8.20	17.22	39.03	0.00	0.34
25-Yr Zero-Coupon Bond	4.68	12.59	79.01	25.00	0.87
10-Yr T-Bond	5.28	3.69	10.53	8.00	0.81
Duration-Based Overlay	Return (%)	Volatility (%)	Drawdown (%)	Duration	Correlation to Liability
Long-Short (Agg. Bond - Cash)	1.32	3.45	11.58	4.34	0.83

Panel2: Liabilities

Net Growth Rate (%)	Benefit Payments to Outstanding Liability (%)	Gross Liability Growth Rate (%)	Duration
1.19	6.06	7.26	12.13

Table 3.3: Summary statistics of data generated by CAP:LINK

Summary statistics of the data generated from CAP:LINK are illustrated (panel 1: core assets and DEO, panel 2: liability). Compared to the historical data, the DEO strategy is modeled in a more conservative fashion (return: 1.3%, volatility: 3.5%, drawdown: 11.6%). In addition, expected annual returns of fixed income securities are lower than the liability growth rate (7.3%) by 1.4 to 2.6%, respectively. The data generated by the CAP:LINK system reflects current issues of the DB pension plan management properly: high performance assets have shorter durations, and long duration assets have lower performance.

[6] We employ OptQuest as the solver, which is embedded in the Crystal Ball software package.

Several points are noteworthy. First, compared to the historical data, the DEO strategy is modeled in a more conservative fashion. Its average return is mere 1.3%, while the historical return is almost four times greater (4.6%). Also, the DEO risk measures (volatility: 3.5%, drawdown: 11.6%) are almost the half the historical values (volatility: 8.3%, drawdown: 19.1%). Next, expected annual returns of fixed income securities are lower than the liability growth rate (7.3%) by 1.4 to 2.6%, respectively. Equities (large-cap & mid-small cap stock indices) are the only asset class whose expected returns exceed the liability growth rate. However, non-fixed income securities are assumed to have zero duration causing relatively low correlations to the liabilities. The data generated by the CAP:LINK system reflects current DB pension plan projections: high performance assets have shorter durations, and long duration assets have lower performance.

Regulatory issues are modeled to approximate actuarial practice. Each year, the sponsoring company must contribute a pre-determined amount of money to the pension plan under certain conditions. To replicate the current 6-year smoothing rule[7], we adopt the following contribution rules.

If $A \geq L$ (or $FR \geq 1$), then contribution = 0
If $0.8L \leq A < L$ (or $0.8 \leq FR < 1$), then contribution = $(L - A)/6$
If $A < 0.8L$ (or $FR < 0.8$), then contribution = $(0.8L - A) + (0.2L)/6$[8]
Where, A: asset, L: liability, and FR: funding ratio = A/L.

Also, we assume the following payout schedules.

1. Benefit payments are paid to retirees at the end of each year.
2. The contributions are made based on the FR right after benefit payments.
3. During the following year, the wealth will change according to performance.

The optimal asset allocations are obtained by solving the following types of optimization problems.

$$\begin{aligned} &\text{Maximize} && W(x_{asset}, x_{DEO}) - \lambda R(x_{asset}, x_{DEO}) \\ &\text{Subject to} && 0 \leq x_{DEO} \leq \gamma \\ & && x_{asset} \geq 0 \text{[9]} \end{aligned}$$

[7] When the funding ratio falls below a threshold level, the sponsoring company is generally obliged to make contributions to make up the deficit within the following 6-year period.
[8] This is to ensure the funding ratio to be always above 0.8.

where, W: performance measures, R: risk measures,
x_{asset}: fixed-mix weights on core assets,
x_{DEO}: fixed-mix weight on DEO,
λ: risk-aversion factor, and γ: maximum DEO.

Several popular performance and risk measures are employed for W and R (Table 4). Efficient frontiers are generated by changing values of λ, and the effect of DEO is evaluated by observing its outcomes with different values of γ.

Performance Measures	Associated Risk Measures
Expected Return	Volatility, Drawdown, Semi-deviation, and Expected Shortfall
Funding Ratio (FR)	Volatility, Min FR, VaR, Expected Shortfall, and Semi-deviation
Wealth	Volatility, VaR, Expected Shortfall, and Semi-deviation
Surplus	Volatility, VaR, Expected Shortfall, and Semi-deviation
Contribution	Volatility, and Maximum Contribution
Economic Value	Volatility, VaR, Expected Shortfall, and Semi-deviation

Table 3.4: List of performance-risk measures for ALM model

5.2. Applying DEO Strategies within an ALM Model

To illustrate the advantages of DEO, we provide an example of ALM models with typical conditions of current DB pension plans in the United States. The plan is assumed to be modestly underfunded at the beginning of the sample period (initial funding ratio = 0.9). The objective is to maximize the economic value at the end of the horizon (T = 10), while controlling the risk of the worst cases. Thus, the ALM model is formulated as follows:

Maximize $\quad EconVal(x_{asset}, x_{DEO}) - \lambda \cdot ES_{5\%}(EconVal(x_{asset}, x_{DEO}))$

Subject to $\quad 0 \le x_{DEO} \le \gamma$

$\quad x_{asset} \ge 0$

where, EconVal = Present Value (Final Surplus – Total Contributions),

$ES_{5\%}(\cdot)$ = Expected Shortfall at 5% level.

[9] In most cases, DB pension plans do not take short positions on core assets except via hedge funds and related asset categories.

Economic value provides a natural objective function for a DB pension plan. It combines the health of the plan and the level of contribution over the planning horizon. Due to the wide set of stakeholders, there are numerous other objective functions for a DB pension plan. Appendix B lists the impact of DEO on several of these objectives. In a similar fashion, DEO strategies improve overall performance.

By maximizing the economic value, we can achieve a healthier plan while reducing excessive contributions to sponsoring companies. Also, as the expected shortfall of the economic value is penalized, the decision rules tend to be more conservative, causing more robust outcomes. Six different levels of maximal DEO (γ = 0, 50, 100, 150, 200 and 250%) are employed to evaluate its effects.

The outcomes from the model are depicted in Figure 3 and Table 5. Indeed, introducing DEO improves performance for the investors possessing long-term goals. As the allowed level of DEO strategies increases, the efficient frontiers in Figure 3 shift to north-west, implying that it is the dominant strategy to apply the DEO. Also, other performance measures can be enhanced by the DEO (Table 5). For instance, as the risk tolerance parameter becomes higher, the final funding ratio increases significantly without causing the rising volatility of the funding ratio.

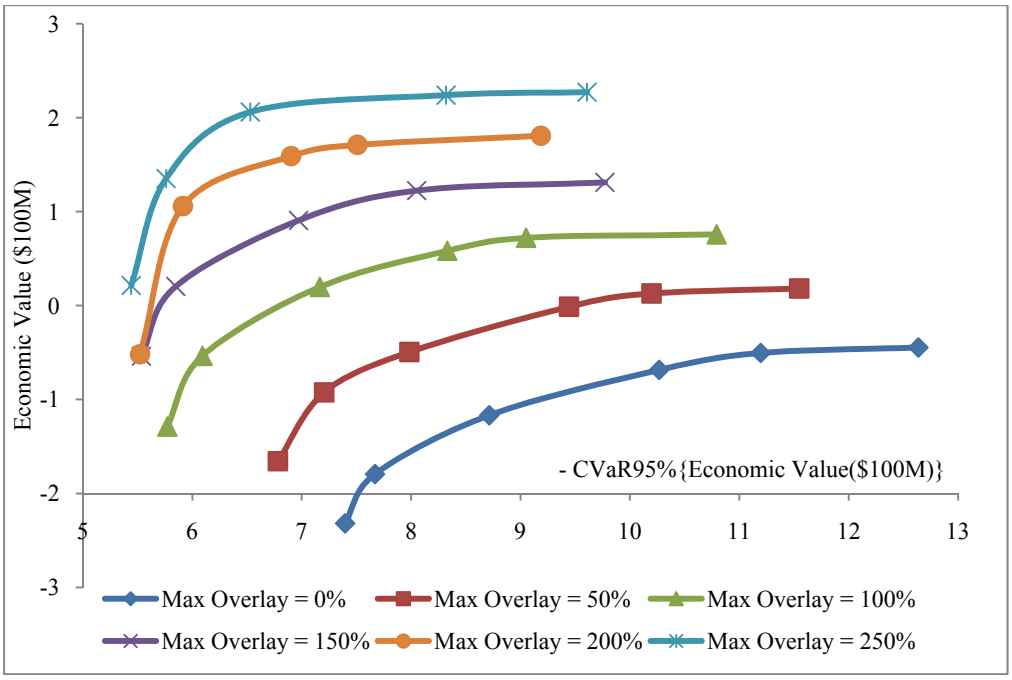

Figure 3: Efficient frontiers of ALM models
An example of ALM models with different levels of DEO strategies (0% to 250%) is illustrated. The economic value (Final Surplus – Total Contributions) and its expected shortfall at 95% level are employed as the performance-risk measure pair. Numbers in red next to the points on efficient frontiers are the duration match ratios. Consistent with the historical test results, applying 100% to 150% of DEO can increase the duration of core assets enough to match that of the liability.

	Max Overlay = 0%				Max Overlay = 100%				Max Overlay = 200%			
	← aggressive		conservative →		← aggressive		conservative →		← aggressive		conservative →	
DM (%)	-100	-38	-7	13	-47	-12	26	35	13	3	37	37
Investment on Assets (%)												
E[R]	8.2	8	7.5	7.1	9.3	9.1	8.5	7.9	10.2	10.3	9.8	8.6
Vol	20.2	16.9	17.9	19.2	22.3	19.6	20.5	20.1	25.1	21	21.6	19.6
DD	38.1	26.2	28.9	33.4	40.5	32.6	36.5	36	49.5	35.3	39.2	33.9
SD	13.7	11.4	11.7	12.3	15.1	13.3	13.6	13.1	17.1	14.5	14.7	13
Funding Ratio (%)												
FR	141	113	103	98	149	125	113	105	158	144	133	112
Vol	69	32	21	16	69	37	25	19	72	51	40	23
$VaR_{95\%}$	76	79	80	79	79	81	83	82	83	84	84	83
$ES_{95\%}$	68	74	75	75	71	75	76	77	73	75	76	78
SD	35	17	11	9	36	21	15	11	39	30	24	14
Min FR	77	82	83	82	79	83	84	84	80	83	84	85
Contribution ($100M)												
Ctbn	3.07	2.04	1.91	2.02	2.48	1.66	1.53	1.58	2.12	1.53	1.43	1.38
Vol	2.89	1.72	1.39	1.24	2.43	1.48	1.16	1.01	2.11	1.48	1.27	1.01
Max	1.45	0.81	0.67	0.66	1.19	0.73	0.62	0.57	1.09	0.76	0.69	0.54
Wealth ($100M)												
Wealth	27.6	22.3	20.6	19.6	29.3	24.7	22.6	20.9	31.4	28.4	26.4	22.2
Vol	13.4	7.4	6.3	5.8	14.7	9	7.4	6.3	16.2	11.4	9.9	6.8
$VaR_{95\%}$	11.7	12.1	11.6	11	12.5	12.8	12.1	11.8	13.3	13.9	13.3	12.6
$ES_{95\%}$	10.3	10.7	9.8	9.2	10.8	11	9.9	9.4	10.9	11.7	11	10
SD	7.4	4.7	4.1	3.9	8.1	5.6	4.8	4.2	9.1	7	6.2	4.5
Surplus ($100M)												
Surplus	3.4	1	0.1	-0.3	4.2	2.1	1	0.3	5.2	3.8	2.8	0.9
Vol	5.7	2.5	1.7	1.4	6.1	3.2	2.2	1.6	6.7	4.4	3.5	2
Economic Value ($100M)												
EV	-0.4	-1.2	-1.8	-2.3	0.8	0.2	-0.5	-1.3	1.8	1.6	1.1	-0.5
Vol	5.8	3.3	2.6	2.2	5.6	3.5	2.7	2.2	5.5	4.1	3.5	2.5
$VaR_{95\%}$	-10.0	-6.9	-6.3	-5.9	-8.2	-5.7	-5.0	-4.9	-6.9	-5.2	-4.6	-4.6
$ES_{95\%}$	-12.6	-8.7	-7.7	-7.4	-10.8	-7.2	-6.1	-5.8	-9.2	-6.9	-5.9	-5.5

Table 5: Performance-risk measures of ALM models
(DM: duration matching ratio, E[R]: expected return, FR: funding ratio, Ctbn: contribution, Vol: volatility, DD: max drawdown, SD: semi-deviation)

Another important feature is the increasing weighted duration match ratio (DM). The weighed duration match ratio is defined as follows.

$$DM(x_{asset}, x_{DEO}) = (\text{Funding Ratio} \cdot D_{asset} - D_{liability})/D_{liability}$$

Since the change of fixed income security returns due to a parallel yield curve shift is -Duration·ΔYield, the change in wealth is -Duration·ΔYield·Wealth. Thus, by controlling DM close to 0, investors can make the change in the total wealth of the core assets from yield changes similar to that of the liability. Similarly, they can control the volatilities on the contributions, which is an importance risk measure for the sponsoring companies.

Since pensions adopting the ALM framework typically suffer from short duration on their core portfolios than liabilities, their main issue is to increase DM. The test results indicate that DEO could increase DM without compromising other performance measures. When reasonably conservative risk aversion factors are adopted, the DM values of core assets with 100% to 150% of DEO are similar to or greater than that of liability (Table 5). It is consistent with the historical test results provided in the previous section; a modest level of DEO strategies can significantly add duration to the core assets. See Appendix B.2, B.4 and B.5 for ALM models that constrain DM explicitly in the problem formulation.

The DEO strategies can help ALM models under many circumstances, yet the details can vary greatly. Also, the CAP:LINK system is designed for long-term risk management. Thus, short-term risk exposures from sharp yield curve changes are not addressed since the system focuses mainly on the long-term investment analytics. As mentioned, the historical data show that an excessive use of DEO can lead to undesirable consequences especially over short time periods. Therefore, careful analysis procedures should be preceded before implementation. See Appendix B for more test results with different risk measures and settings.

6. Conclusions and Future Research

We have shown that for investors possessing a set of long-term liabilities and future contributions such as DB pension plans, there is much to gain by implementing specialized overlay strategies (in careful doses). Both the empirical evidence with historical returns and the forward looking tests illustrate the advantages of these concepts. The added duration reduces contribution risks while increases expected portfolio performance.

What are possible next steps? First, in order to distinguish the effects of DEO from the traditional leverage, the ALM model should consider the practical issues upon implementing them such as fees for

leverage, limits on overlay based on the core portfolio characteristics, etc. While the traditional leverage could be less efficient due to the liquidity and the transaction costs, it would be meaningful to identify the usefulness of DEO over the leverage.

Second, short-term effects of DEO should be more carefully analyzed. The main weakness comes from the potential disruption of core asset performance due to the yield curve changes which typically occurs in relatively short time periods, so further analysis via high-frequency data is essential. The recent turmoil in the global financial market could provide a good data source for the empirical tests. Ideally, a dynamic strategy can be developed to improve both short term and long term results. To this end, the investor must be able to execute the overlay strategy in a timely fashion. Other duration enhancing strategies should be evaluated, beyond the fixed-mix and duration-matching rules which have been tested in this report.

Another topic involves combining the duration-based strategy with overlays in other domains, for example, commodities and currencies. Previous research has shown the advantages of diverse overlays (Mulvey, Ural, and Zhang (2007)). The goal in these studies is to improve risk adjusted performance by increasing diversification for the asset portfolios and the ALM portfolios. Wide diversification can lead to higher returns, especially for long-term investors. Again, it is important to implement the strategies with highly liquid assets so that rebalancing can be accomplished efficaciously in concert with careful risk management.

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Appendix A. Test results with 30-yr government bond index as a proxy for liabilities

Appendix A.1. Performance of key strategies over 1982 to 2007 with 30-yr bond index as liability

1. Asset-Only Performance Measures

	Liability	Equity	Bond	Tbill	60-40	70-30	DEO
Annual Return	10.02%	13.06%	10.24%	5.38%	12.26%	12.50%	4.63%
Volatility	11.13%	14.66%	8.38%	0.70%	9.88%	10.93%	8.33%
Maximal Drawdown	18.44%	44.73%	11.64%	0.00%	20.80%	27.45%	19.06%
Sharpe Ratio	0.418	0.524	0.584	N/A	0.699	0.653	-0.090
Excess return/Drawdown	0.252	0.172	0.418	N/A	0.331	0.259	-0.039

2. Short-term risk measures

	Liability	Equity	Bond	Tbill	60-40	70-30	DEO
Expected Shortfall at 5% Level	-5.84%	-8.98%	-4.09%	0.08%	-5.12%	-6.03%	-4.55%
Expected Shortfall at 1% Level	-7.69%	-14.00%	-5.33%	0.08%	-7.56%	-9.00%	-5.77%
Return for Worst Month	-10.99%	-21.53%	-6.71%	0.07%	-10.45%	-13.22%	-6.79%

3. Relative Performance Measures

	Liability	Equity	Bond	Tbill	60-40	70-30	DEO
Correlation to Liability	1.000	0.159	0.947	0.065	0.462	0.367	0.947
Tracking Error	0.00%	3.03%	0.22%	-4.64%	2.24%	2.48%	-5.39%
Std(Tracking Error)	0.00%	16.94%	4.19%	11.11%	10.95%	12.42%	4.21%
Information Ratio	N/A	0.179	0.052	-0.418	0.204	0.200	-1.280

Appendix A.2. Performance of 60-40 strategies after applying DEO with 30-yr bond index as liability

1. Asset Performance Measures

DEO (%)	0	25	50	75	100	125	150	175	200
Annual Return (%)	12.2	13.5	14.7	15.8	16.9	18.0	19.1	20.1	21.0
Volatility (%)	9.8	11.0	12.4	13.9	15.6	17.4	19.2	21.1	23.0
Maximal Drawdown (%)	20.8	16.8	16.5	20.6	25.0	29.1	33.1	36.9	40.6
Sharpe Ratio	0.69	0.73	0.75	0.75	0.74	0.73	0.71	0.69	0.68
Excess return/Max. Drawdown	0.33	0.48	0.56	0.50	0.46	0.43	0.41	0.39	0.38

2. Short-term risk measures

DEO (%)	0	25	50	75	100	125	150	175	200
Return for 5 Worst Month (%)	-5.1	-5.4	-6.1	-6.9	-7.8	-8.9	-10.0	-11.1	-12.2
Return for 1 Worst Month (%)	-7.5	-7.6	-8.1	-9.0	-10.0	-11.2	-12.5	-13.8	-15.1
Return for Worst Month (%)	-10.5	-9.0	-8.8	-10.4	-12.1	-13.7	-15.3	-16.9	-18.6

3. Relative Performance Measures

DEO (%)	0	25	50	75	100	125	150	175	200
Correlation to Liability	0.46	0.54	0.68	0.75	0.79	0.82	0.85	0.87	0.88
Tracking Error (%)	2.2	3.4	4.6	5.8	6.9	8.0	9.0	10.1	11.0
Std(Tracking Error) (%)	10.9	9.9	9.3	9.2	9.5	10.2	11.3	12.6	14.2
Information Ratio	0.20	0.34	0.49	0.63	0.72	0.78	0.80	0.79	0.77

Appendix A.3. Performance of 70-30 strategies after applying DEO with 30-yr bond index as liability

1. Asset Performance Measures

DEO (%)	0	25	50	75	100	125	150	175	200
Annual Return (%)	12.5	13.7	14.9	16.1	17.3	18.4	19.4	20.4	21.4
Volatility (%)	10.9	11.8	13.0	14.4	16.0	17.6	19.4	21.2	23.0
Maximal Drawdown (%)	27.4	23.4	19.6	20.1	24.4	28.6	32.6	36.4	40.1
Sharpe Ratio	0.65	0.70	0.73	0.74	0.74	0.73	0.72	0.71	0.69
Excess return/Max. Drawdown	0.25	0.35	0.48	0.53	0.48	0.45	0.43	0.41	0.40

2. Short-term risk measures

DEO (%)	0	25	50	75	100	125	150	175	200
Return for 5 Worst Month (%)	-6.0	-6.1	-6.5	-7.1	-8.0	-8.9	-9.9	-11.0	-12.1
Return for 1 Worst Month (%)	-9.0	-8.8	-9.1	-9.5	-10.4	-11.4	-12.5	-13.8	-15.0
Return for Worst Month (%)	-13.2	-11.7	-10.3	-10.4	-12.0	-13.7	-15.3	-16.9	-18.5

3. Relative Performance Measures

DEO (%)	0	25	50	75	100	125	150	175	200
Correlation to Liability	0.36	0.50	0.60	0.68	0.74	0.78	0.81	0.84	0.85
Tracking Error (%)	2.4	3.7	4.9	6.1	7.2	8.3	9.4	10.4	11.4
Std(Tracking Error) (%)	12.4	11.4	10.8	10.5	10.7	11.2	12.1	13.3	14.6
Information Ratio	0.20	0.32	0.45	0.57	0.67	0.74	0.77	0.78	0.78

Appendix B. Additional results on empirical tests with ALM models

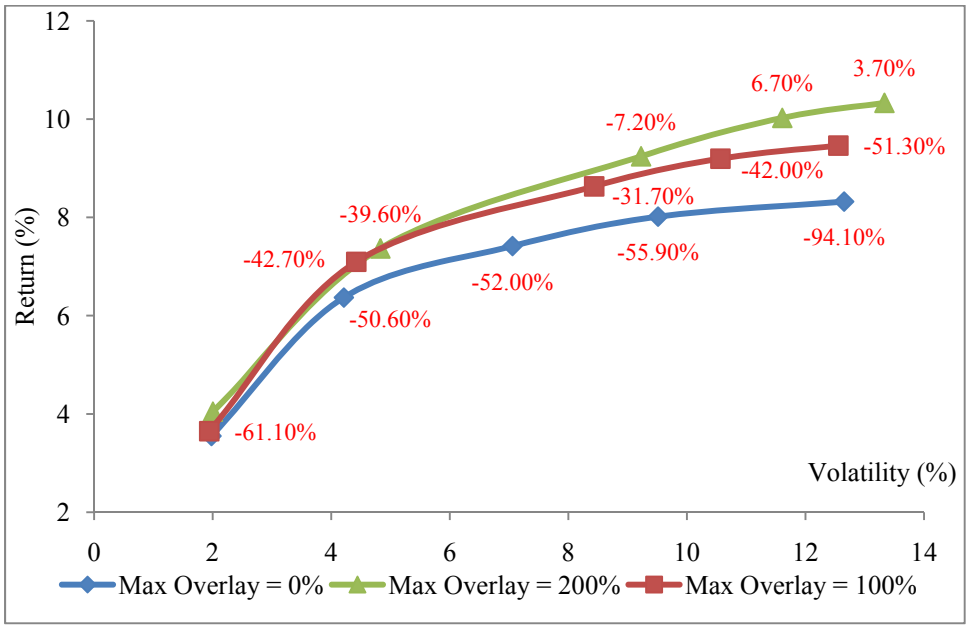
We discuss the positive effects of DEO using various performance-risk measures. The main results indicate that employing a moderate level of DEO can improve the performance-risk trade-off as in section 5.

Importantly, institutional managers tend to focus on shorter term performance since their performance is evaluated based on short- to mid-term investment outcomes, although the ultimate objective is to achieve long term goals. Therefore, in order to ensure a successful implementation of DEO in practice, it is imperative to examine the investment outcomes with a shorter time horizon as well. Fortunately, the test results show that one can still achieve superior investment performance by applying a moderate level of DEO for a shorter time horizon (T=5).

Appendix B.1. Additional ALM model (1)

The return and the volatility of core assets are employed as the performance-risk measure pair.

Numbers in red next to the points on efficient frontiers are the DM values.



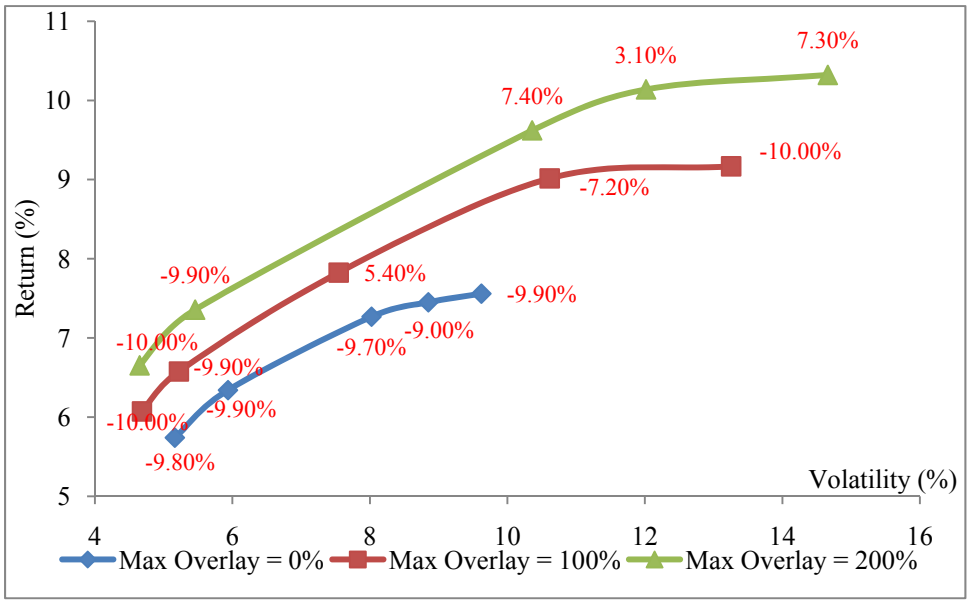
Appendix B.1. (continued) Additional ALM model (1)

	Max Overlay = 0%				Max Overlay = 100%				Max Overlay = 200%			
	← aggressive			conservative →	← aggressive			conservative →	← aggressive			conservative →
Investment on Assets (%)												
E[R]	8.3	7.4	6.4	3.6	9.5	8.6	7.1	3.6	10.3	9.2	7.4	4
Vol	16	11.8	8.4	4.4	17.4	13.8	9	3.9	20	15.8	9.9	4.5
DD	24.4	13.4	7.3	3.5	25.2	16.9	8.1	2.3	31.8	21.6	9.7	2.9
SD	11	8	5.7	3	12	9.5	6.1	2.6	13.8	10.9	6.7	3
Funding Ratio (%)												
FR	128	104	95	87	136	115	99	88	145	121	101	88
Vol	54	34	27	19	54	35	24	19	55	34	24	19
$VaR_{95\%}$	77	78	77	72	79	81	79	71	82	81	79	72
$ES_{95\%}$	71	75	75	69	74	76	76	68	75	78	76	69
SD	26	13	9	7	28	17	10	8	31	19	11	8
Min FR	80	84	82	74	82	85	83	74	83	85	84	75
Contribution ($100M)												
Ctbn	2.46	1.97	2.51	4.97	1.85	1.53	2.10	4.93	1.52	1.34	1.95	4.57
Vol	2.29	1.57	1.70	2.56	1.82	1.35	1.56	2.61	1.50	1.17	1.50	2.53
Max	1.14	0.62	0.72	1.58	0.85	0.55	0.65	1.62	0.72	0.51	0.58	1.49
Wealth ($100M)												
Wealth	24.9	20	18.4	17.1	26.5	22.3	19.2	17.2	28.4	23.6	19.5	17.3
Vol	9.2	4.8	3.9	4.2	10	6	3.9	4.2	11.6	6.8	3.9	4.2
$VaR_{95\%}$	13.1	13.1	12.9	10.8	13.3	13.7	13.6	10.8	13.8	14	13.8	11.2
$ES_{95\%}$	11.2	11.8	11.9	10	11.8	12.4	12.5	10.2	11.8	12	12.6	10.5
SD	5.6	3.1	2.5	2.8	6	3.9	2.6	2.8	7	4.4	2.6	2.7
Surplus ($100M)												
Surplus	2.1	-0.1	-0.9	-1.5	2.9	0.9	-0.5	-1.4	3.8	1.5	-0.4	-1.4
Vol	3.7	1.7	1.3	1.3	4.1	2.2	1.4	1.3	4.6	2.5	1.5	1.3
Economic Value ($100M)												
EV	-0.7	-2.1	-3.4	-6.5	0.5	-0.7	-2.6	-6.4	1.6	0.1	-2.3	-6.0
Vol	4.4	2.9	2.7	3.4	4.3	3.0	2.7	3.5	4.2	3.1	2.7	3.4
$VaR_{95\%}$	-8.0	-6.9	-8.1	-12.0	-6.8	-5.7	-7.2	-12.2	-5.5	-4.9	-7.0	-11.6
$ES_{95\%}$	-10.6	-8.6	-9.9	-14.7	-8.8	-7.4	-9.2	-14.8	-7.4	-6.5	-8.9	-14.2

E[R]: expected return, FR: funding ratio, Ctbn: contribution, Vol: volatility, DD: max drawdown, SD: semi-deviation

Appendix B.2. Additional ALM model (2)

The return and the volatility of core assets are employed as the performance-risk measure pair. A constraint on duration matching ratio is introduced (DM ≥ 0.9). Numbers in red next to the points on efficient frontiers are the DM values.



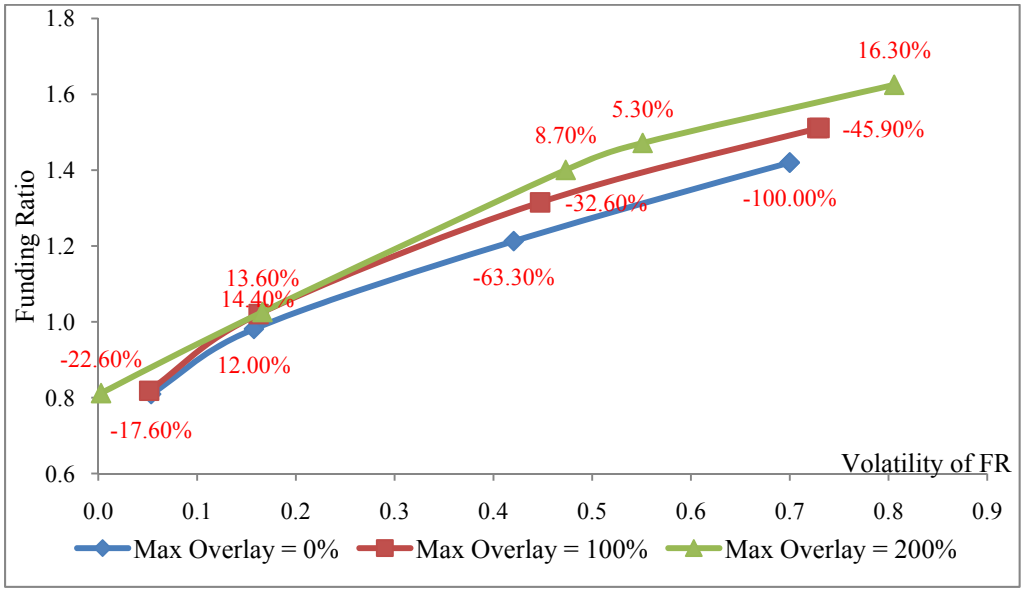
Appendix B.2. (continued) Additional ALM model (2)

	Max Overlay = 0%				Max Overlay = 100%				Max Overlay = 200%			
	← aggressive			conservative →	← aggressive			conservative →	← aggressive			conservative →
Investment on Assets (%)												
E[R]	7.6	7.3	6.3	5.7	9.2	7.8	6.6	6.1	10.3	9.6	7.4	6.7
Vol	17.7	16.1	13.2	12.5	20.3	15	11.9	11.5	20.7	17.1	12	11.1
DD	28.5	23.7	17.4	16.9	34	20.8	14.5	14.5	33.6	24.9	14.8	13.5
SD	11.6	10.4	8.5	8.1	13.7	9.9	7.8	7.6	14.1	11.8	8.2	7.5
Funding Ratio (%)												
FR	104	98	91	87	131	102	92	90	150	127	98	95
Vol	21	18	12	11	45	20	15	11	65	37	17	15
$VaR_{95\%}$	79	80	78	77	80	80	79	77	81	82	80	78
$ES_{95\%}$	75	77	76	75	75	77	76	74	74	78	77	75
SD	12	9	6	5	25	11	7	6	35	22	9	8
Min FR	83	84	83	82	82	86	84	82	82	85	85	82
Contribution ($100M)												
Ctbn	1.83	1.78	2.26	2.67	1.84	1.57	2.17	2.51	1.72	1.31	1.79	2.27
Vol	1.34	1.17	1.23	1.35	1.74	1.13	1.29	1.37	1.75	1.17	1.21	1.44
Max	0.64	0.49	0.53	0.56	0.82	0.44	0.51	0.63	0.81	0.54	0.50	0.67
Wealth ($100M)												
Wealth	20.6	19.4	17.9	17.3	25.9	20.1	18.1	17.7	29.4	24.8	19.2	18.6
Vol	6.3	5.2	4.5	4.2	10.4	5.1	4.2	4.2	13.2	8	4.3	4.2
$VaR_{95\%}$	11.6	11.5	11	11.3	13	12.7	11.8	11.7	13.8	13.7	12.9	12.5
$ES_{95\%}$	9.9	10.1	9.5	9.4	11	10.4	10.2	9.7	11.5	11.8	11.1	10.7
SD	4.1	3.5	3	2.8	6.2	3.4	2.8	2.8	7.6	5.1	2.9	2.8
Surplus ($100M)												
Surplus	0.2	-0.4	-1.1	-1.4	2.6	-0.1	-1	-1.2	4.2	2.1	-0.5	-0.8
Vol	1.8	1.4	1	1	4	1.6	1.1	1.1	5.5	2.9	1.3	1.3
Economic Value ($100M)												
EV	-1.7	-2.2	-3.3	-4.1	0.4	-1.7	-3.2	-3.7	1.6	0.6	-2.3	-3.1
Vol	2.6	2.2	2.1	2.2	4.2	2.4	2.2	2.2	4.8	3.2	2.3	2.4
$VaR_{95\%}$	-6.0	-6.0	-6.6	-7.5	-6.3	-5.5	-6.8	-7.5	-6.3	-4.6	-5.9	-6.9
$ES_{95\%}$	-7.4	-7.4	-8.4	-9.4	-8.4	-7.2	-8.7	-9.4	-8.6	-6.2	-7.9	-9.2

E[R]: expected return, FR: funding ratio, Ctbn: contribution, Vol: volatility, DD: max drawdown, SD: semi-deviation

Appendix B.3. Additional ALM model (3)

The funding ratio and its volatility are employed as the performance-risk measure pair. Numbers in red next to the points on efficient frontiers are the DM values.



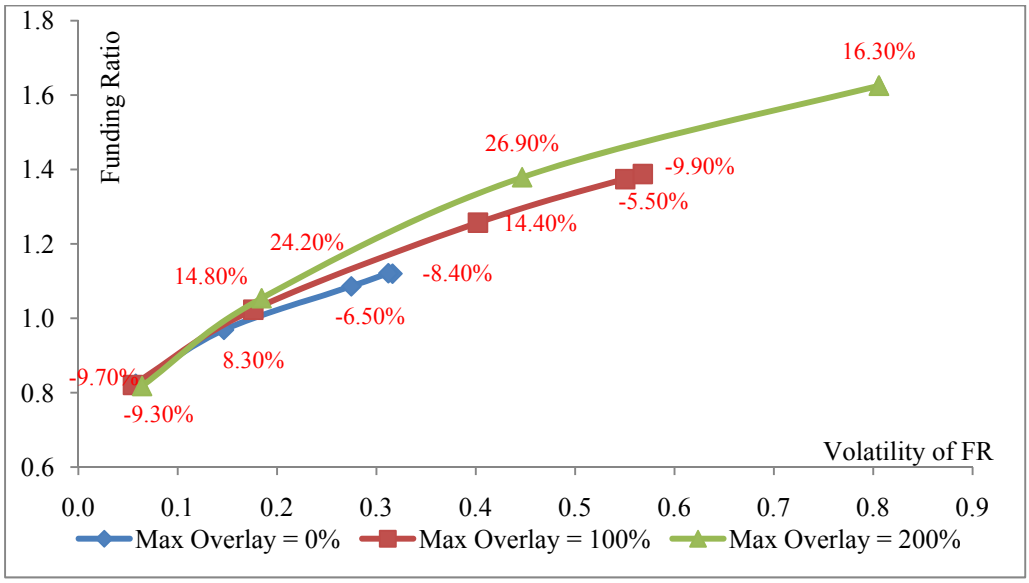
Appendix B.3. (continued) Additional ALM model (3)

	Max Overlay = 0%				Max Overlay = 100%				Max Overlay = 200%			
	← aggressive			conservative →	← aggressive			conservative →	← aggressive			conservative →
Investment on Assets (%)												
E[R]	8.2	8.2	7.1	2.4	9.3	9.3	7.8	3.3	10.1	10.1	7.9	2.6
Vol	20.5	16.9	19	11.4	22.8	19.7	17.5	12.2	26.1	21.4	17.3	10.8
DD	39	26.5	33	24	42.5	32.7	27.8	24.2	53.5	37.4	27.2	22.3
SD	13.9	11.6	12.2	7.2	15.5	13.5	11.6	7.9	17.7	14.8	11.5	7.1
Funding Ratio (%)												
FR	142	121	98	81	151	132	102	82	163	140	103	81
Vol	70	42	16	5	73	45	16	5	81	47	17	5
$VaR_{95\%}$	75	78	79	74	78	80	81	75	80	83	82	74
$ES_{95\%}$	67	72	75	72	70	74	78	74	72	75	78	73
SD	36	22	9	3	38	25	10	3	42	28	10	3
Min FR	77	81	82	76	79	82	85	78	79	83	85	76
Contribution ($100M)												
Ctbn	3.14	2.20	1.93	5.67	2.59	1.83	1.52	4.81	2.35	1.54	1.49	5.52
Vol	3.00	2.01	1.13	2.10	2.58	1.71	1.00	1.95	2.37	1.44	0.99	2.25
Max	1.49	0.97	0.62	1.18	1.25	0.84	0.48	0.96	1.20	0.76	0.47	1.12
Wealth ($100M)												
Wealth	27.8	23.8	19.6	16.2	29.8	26	20.3	16.4	32.4	27.8	20.4	16.3
Vol	13.7	8.6	5.8	4.4	15.5	10.1	5.8	4.4	18.1	11	5.8	4.4
$VaR_{95\%}$	11.7	12.4	11.2	9.7	12.6	13	11.7	9.7	13.2	13.4	11.9	9.7
$ES_{95\%}$	10.2	11.1	9.2	7.8	10.6	11.4	9.7	8	10.7	11.5	9.8	7.9
SD	7.5	5.2	3.9	2.9	8.4	6.2	3.9	3	9.7	6.8	3.9	2.9
Surplus ($100M)												
Surplus	3.5	1.6	-0.3	-1.9	4.4	2.6	0	-1.8	5.6	3.5	0.1	-1.9
Vol	5.8	3.2	1.4	0.9	6.5	3.8	1.4	0.9	7.6	4.1	1.4	0.9
Economic Value ($100M)												
EV	-0.5	-0.8	-2.3	-7.6	0.8	0.5	-1.5	-6.6	1.8	1.4	-1.4	-7.4
Vol	5.9	4.0	2.1	2.8	5.9	4.0	2.1	2.7	6.1	3.9	2.1	3.0
$VaR_{95\%}$	-10.3	-7.4	-5.8	-12.0	-8.5	-6.0	-5.0	-10.8	-7.4	-5.1	-4.9	-12.0
$ES_{95\%}$	-12.8	-9.5	-6.8	-14.2	-11.2	-8.0	-6.0	-13.0	-10.1	-6.7	-6.0	-14.4

E[R]: expected return, FR: funding ratio, Ctbn: contribution, Vol: volatility, DD: max drawdown, SD: semi-deviation

Appendix B.4. Additional ALM model (4)

The funding ratio and its volatility are employed as the performance-risk measure pair. A constraint on duration matching ratio is introduced (DM ≥ 0.9). Numbers in red next to the points on efficient frontiers are the DM values.



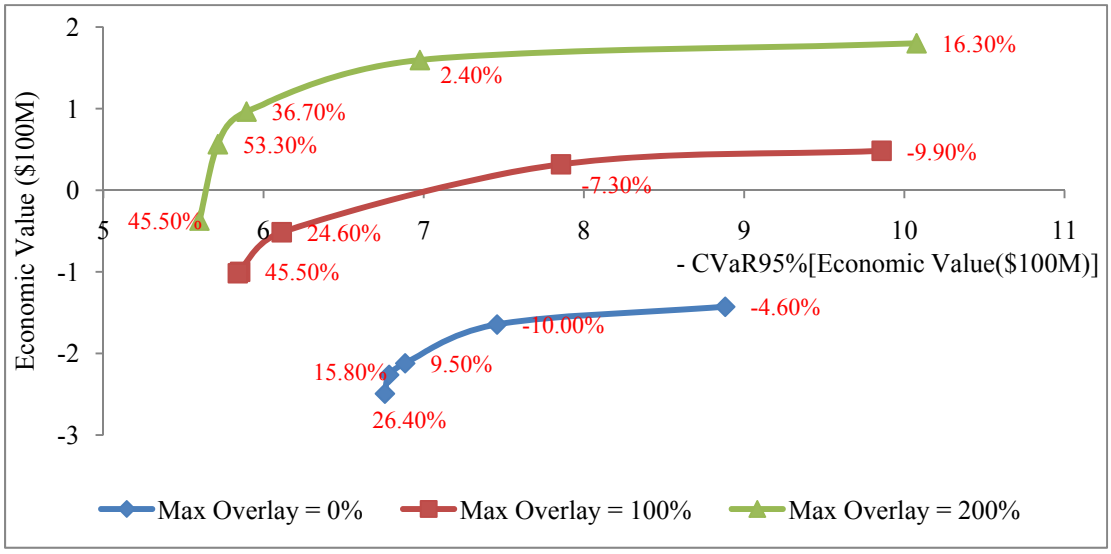
Appendix B.4. (continued) Additional ALM model (4)

	Max Overlay = 0%				Max Overlay = 100%				Max Overlay = 200%			
	← aggressive			conservative →	← aggressive			conservative →	← aggressive			conservative →
Investment on Assets (%)												
E[R]	7.7	7.6	7	3.6	9.1	8.9	7.8	3.4	10.1	9.9	8.2	2.4
Vol	19	18.4	18.3	13.6	22.2	21.4	19	13.2	26.1	22.5	17.3	12.7
DD	32	30.2	30.7	26.6	40.5	38.2	32.4	26.2	53.5	41.7	27.6	30.9
SD	12.4	12	11.7	8.6	14.9	14.3	12.5	8.4	17.7	15.4	11.6	8.5
Funding Ratio (%)												
FR	112	109	97	82	137	126	102	82	163	138	105	82
Vol	31	28	15	6	55	40	18	6	81	45	19	6
$VaR_{95\%}$	77	78	79	75	80	81	81	75	80	83	82	73
$ES_{95\%}$	74	74	76	74	73	75	77	74	72	75	79	71
SD	17	15	8	3	30	23	10	3	42	26	11	4
Min FR	82	82	83	78	81	82	84	78	79	83	85	76
Contribution ($100M)												
Ctbn	2.13	2.08	1.92	4.52	2.19	1.84	1.59	4.70	2.35	1.57	1.40	5.66
Vol	1.80	1.69	1.10	1.81	2.12	1.67	1.05	1.87	2.37	1.43	0.96	2.40
Max	0.82	0.76	0.59	0.90	1.00	0.80	0.51	0.94	1.20	0.78	0.46	1.19
Wealth ($100M)												
Wealth	22.3	21.6	19.3	16.5	27.3	25.1	20.4	16.4	32.4	27.5	20.9	16.4
Vol	8	7.3	5.6	4.4	12.7	10.1	5.9	4.4	18.1	11.1	5.8	4.4
$VaR_{95\%}$	11.8	11.6	11.3	9.9	12.4	12.5	11.6	9.9	13.2	13	12.3	9.5
$ES_{95\%}$	10	9.9	9.2	8	10.5	10.4	9.8	8	10.7	11	10	7.9
SD	4.9	4.6	3.7	3	7.2	6	3.9	3	9.7	6.8	3.9	3
Surplus ($100M)												
Surplus	0.9	0.6	-0.4	-1.8	3.3	2.2	0	-1.8	5.6	3.3	0.3	-1.8
Vol	2.7	2.4	1.3	0.9	5	3.7	1.5	0.9	7.6	4.1	1.6	1
Economic Value ($100M)												
EV	-1.3	-1.5	-2.4	-6.3	0.4	0.1	-1.6	-6.5	1.8	1.3	-1.1	-7.5
Vol	3.6	3.4	2.0	2.5	4.9	4.0	2.2	2.6	6.1	3.9	2.2	3.1
$VaR_{95\%}$	-7.0	-6.8	-5.8	-10.3	-7.4	-6.2	-5.2	-10.6	-7.4	-5.2	-4.8	-12.6
$ES_{95\%}$	-9.1	-8.8	-6.9	-12.2	-9.7	-8.1	-6.2	-12.6	-10.1	-6.5	-5.8	-14.8

E[R]: expected return, FR: funding ratio, Ctbn: contribution, Vol: volatility, DD: max drawdown, SD: semi-deviation

Appendix B.5. Additional ALM model (5)

The economic value (Final Surplus – Total Contributions) and its expected shortfall at 95% level are employed as the performance-risk measure pair. A constraint on duration matching ratio is introduced (DM ≥ 0.9). Numbers in red next to the points on efficient frontiers are the DM values.



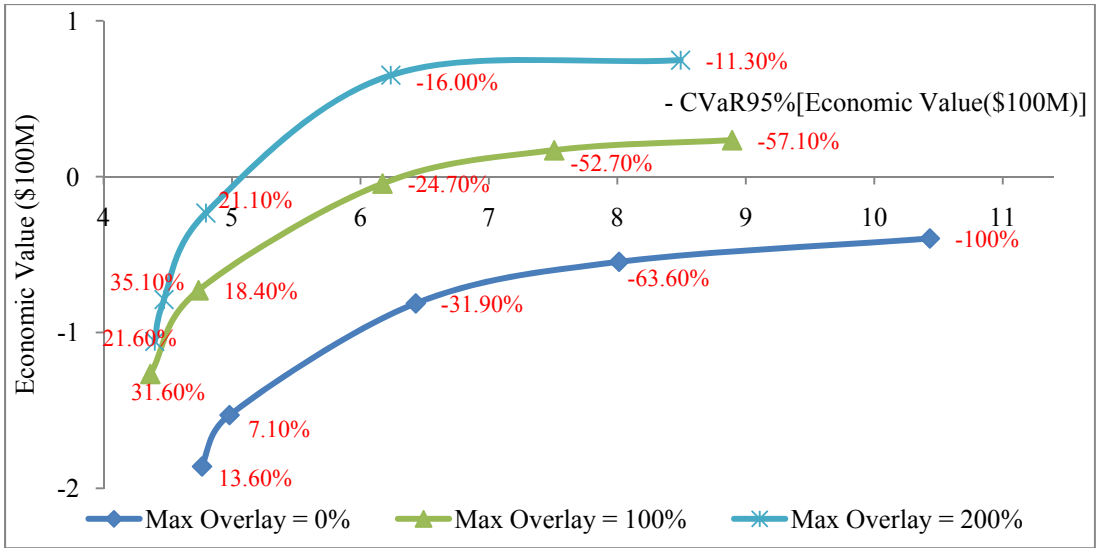
Appendix B.5. (continued) Additional ALM model (5)

	Max Overlay = 0%				Max Overlay = 100%				Max Overlay = 200%			
	← aggressive			conservative →	← aggressive			conservative →	← aggressive			conservative →
Investment on Assets (%)												
E[R]	7.7	7.2	7	6.8	9.1	8.5	7.9	7.4	10.1	9.8	9.3	8.7
Vol	18.9	19.1	19.6	20.5	22.2	20.5	21	17.6	26.1	20.8	22.5	20.4
DD	31.9	33	34.9	38.4	40.5	36.2	38.8	28.1	53.5	36.4	43.3	37
SD	12.4	12.3	12.5	13	15	13.6	13.6	11.4	17.7	14.1	15.1	13.6
Funding Ratio (%)												
FR	110	99	98	97	139	113	107	100	163	131	126	114
Vol	29	17	16	16	57	25	21	18	81	39	34	25
$VaR_{95\%}$	78	79	79	78	79	83	81	79	80	85	84	83
$ES_{95\%}$	74	75	75	74	73	76	76	76	72	77	76	77
SD	16	10	9	9	31	15	12	10	42	23	21	15
Min FR	82	82	82	81	80	84	83	84	79	84	83	84
Contribution ($100M)												
Ctbn	2.10	1.91	1.97	2.10	2.23	1.53	1.65	1.77	2.35	1.37	1.48	1.41
Vol	1.74	1.17	1.13	1.08	2.18	1.16	1.10	1.17	2.37	1.22	1.24	1.06
Max	0.79	0.64	0.65	0.69	1.03	0.62	0.62	0.56	1.20	0.64	0.70	0.58
Wealth ($100M)												
Wealth	21.9	19.8	19.7	19.5	27.6	22.6	21.3	20	32.4	26	25.1	22.6
Vol	7.6	5.9	5.9	5.8	13	7.5	6.7	5.8	18.1	9.4	9	7.1
$VaR_{95\%}$	11.7	11.3	11	10.9	12.4	12.1	11.8	11.7	13.2	13.4	13	12.6
$ES_{95\%}$	10	9.3	9.1	8.8	10.5	10	9.4	9.8	10.7	11	10.3	10.1
SD	4.7	4	3.9	3.9	7.3	4.9	4.4	3.8	9.7	6	5.8	4.7
Surplus ($100M)												
Surplus	0.8	-0.2	-0.3	-0.4	3.4	1.1	0.5	-0.2	5.6	2.7	2.2	1.1
Vol	2.6	1.5	1.4	1.4	5.2	2.2	1.8	1.5	7.6	3.4	3	2.1
Economic Value ($100M)												
EV	-1.4	-2.1	-2.3	-2.5	0.5	-0.5	-1.0	-1.0	1.8	1.0	0.6	-0.4
Vol	3.5	2.2	2.1	1.9	5.1	2.7	2.3	2.2	6.1	3.4	3.2	2.6
$VaR_{95\%}$	-6.9	-5.6	-5.6	-5.8	-7.5	-4.9	-5.0	-5.6	-7.4	-4.6	-4.6	-4.8
$ES_{95\%}$	-8.9	-6.9	-6.8	-6.8	-9.9	-6.1	-5.9	-5.8	-10.1	-5.9	-5.7	-5.6

E[R]: expected return, FR: funding ratio, Ctbn: contribution, Vol: volatility, DD: max drawdown, SD: semi-deviation

Appendix B.6. (continued) Additional ALM model (6)

An example of ALM models with different levels of DEO strategies (0% to 250%) is illustrated for a shorter time period (T=5). The economic value (Final Surplus – Total Contributions) and its expected shortfall at 95% level are employed as the performance-risk measure pair. Numbers in red next to the points on efficient frontiers are the DM values. Consistent with the historical test results with a long term horizon, applying 100% to 150% of DEO can increase the duration of core assets enough to match that of the liability for a shorter period.



Appendix B.6. (continued) Additional ALM model (6)

	Max Overlay = 0%				Max Overlay = 100%				Max Overlay = 200%			
	← aggressive			conservative →	← aggressive			conservative →	← aggressive			conservative →
Investment on Assets (%)												
E[R]	8.46	7.77	6.68	6.21	9.33	8.88	7.81	7.03	9.94	8.58	7.73	7.35
Vol	18.7	16.5	17.8	17.6	20.9	18.4	19.5	19.5	24.7	19.4	20	19.6
DD	18.9	14.2	16.7	16.8	21.3	16.4	18.6	19.4	28.3	18	19.8	19.4
SD	11.9	10.4	10.8	10.7	13.3	11.7	12	11.9	15.7	12.1	12.4	12.1
Funding Ratio (%)												
FR	118	104	96	94	120	110	102	98	124	106	101	99
Vol	36	17	9	7	34	20	12	9	37	14	11	10
$VaR_{95\%}$	1	1	1	1	1	1	1	1	1	1	1	1
$ES_{95\%}$	1	1	1	1	1	1	1	1	1	1	1	1
SD	0	0	0	0	0	0	0	0	0	0	0	0
Min FR	81	86	87	87	83	86	87	87	82	87	87	87
Contribution ($100M)												
Ctbn	1.90	1.16	1.04	1.09	1.59	1.10	0.94	0.98	1.55	0.90	0.94	0.95
Vol	1.96	0.97	0.54	0.44	1.64	0.99	0.62	0.48	1.61	0.64	0.56	0.50
Max	1.10	0.53	0.38	0.35	0.91	0.56	0.41	0.38	0.92	0.43	0.41	0.39
Wealth ($100M)												
Wealth	20.2	18	16.7	16.3	20.7	19.1	17.8	17	21.7	18.4	17.7	17.3
Vol	6.48	4.37	3.83	3.61	7.07	5.11	4.44	4	8.32	4.59	4.27	4.08
$VaR_{95\%}$	12	11.4	10.6	10.5	11.8	11.6	11	10.9	11.1	11.4	11.3	11.2
$ES_{95\%}$	10.7	10.5	9.49	9.42	10.5	10.5	9.55	9.44	9.92	9.97	9.6	9.55
SD	3.86	2.89	2.62	2.49	4.22	3.31	3	2.76	4.9	3.11	2.93	2.82
Surplus ($100M)												
Surplus	1.89	0.37	-0.5	-0.8	2.25	1.13	0.22	-0.3	2.88	0.67	0.15	-0.1
Vol	3.94	1.96	1.08	0.89	4.02	2.39	1.43	1.03	4.62	1.66	1.28	1.12
Economic Value ($100M)												
EV	-0.4	-0.8	-1.5	-1.9	0.2	0.0	-0.7	-1.3	0.7	-0.2	-0.8	-1.1
Vol	4.5	2.6	1.5	1.2	4.2	2.9	1.8	1.3	4.5	2.1	1.6	1.4
$VaR_{95\%}$	-7.8	-5.0	-3.9	-3.7	-6.7	-4.8	-3.8	-3.5	-6.3	-3.6	-3.6	-3.5
$ES_{95\%}$	-10.0	-6.0	-4.6	-4.4	-8.5	-5.8	-4.3	-4.0	-8.1	-4.4	-4.1	-4.0

E[R]: expected return, FR: funding ratio, Ctbn: contribution, Vol: volatility, DD: max drawdown, SD: semi-deviation