Linking Momentum Strategies with Single-Period Portfolio Models

John M. Mulvey, Woo Chang Kim, and Mehmet Bilgili
Princeton University

March 12, 2008

Abstract

Several versions of the Markowitz portfolio model are evaluated with respect to patterns in equity markets. Much research has shown that strategies based on momentum have generated superior risk adjusted returns. We form a long-only portfolio of momentum strategies via industry-level assets; the strategy beats many others over numerous markets and time periods and provides a good benchmark for competing optimization models. Simple Markowitz models are quite effective, as long as the proper historical time period is chosen for the stochastic projections. Investment performance of optimal asset allocation models can be improved by considering the momentum effects in the parameter estimation procedures.

I. Introduction

Since the early work of Markowitz (1952, 1956), the single-period mean-variance model has been the norm in portfolio management. Over the years, numerous variants of the core model have been proposed and implemented to improve performance in practical settings. However, empirical tests suggest that investment performance has not been outstanding in many cases. In this paper, we illustrate examples to increase the investment performance of such models by actively exploiting a significant market anomaly – momentum effects.

The most persistent equity anomaly involves the predictability of stock returns based on past performance, which is often referred to as the momentum effect. The typical investment strategy in academic articles that exploits the effect is to buy winners and sell losers based on the intermediate term performance (3 to 12 months) proposed by Jegadeesh and Titman (1993). Many papers such as Cleary et al. (1998), Rouwenhorst (1998), Kang et al. (2002), and Demir et al. (2004) have documented that such a strategy is profitable, except a few stock markets (Liu and Lee (2001) and Hameed and Kusnadi (2002)). Further, money managers, dominant players in the stock market, are reported to not only employ the momentum effect, but also improve their performance by applying it. For instance, Grinblatt et al. (1995), Nofsinger and Sias (1999), Sias et al. (2001), Badrinath and Wahal (2002), Sapp and Tiwari (2004), and Sias (2004) document that a significant proportion of active funds adopt the momentum strategy as their equity selection rules. Carhart (1997) illustrates that the performance persistence of mutual funds can be explained by the one-year momentum effect. Recently, Mulvey and Kim (2008) show that the active equity funds in the U.S. share similar performance patterns with the industry-level long-only momentum strategy, and the similarity is stronger for the funds with good performance. Thus, we apply the specialized momentum strategy as a basis, and benchmark for single-period optimization models in the equity domain.

Unlike traditional approaches, we adopt industry-level data for the empirical analysis. Why do we employ industry-level data? First, compared to stock-level analysis, it reduces idiosyncratic risks. Since the mean-variance models require the estimated market parameters, it may lead to unstable test results to adopt stock-level data without employing specialized parameter estimation techniques. In contrast, one can readily eliminate such issues by analyzing the broad asset classes such as industries. Second, the strategy is becoming easy to implement due to the introduction of various exchange traded funds (ETFs). In addition, the industry-level momentum strategy has displayed outstanding performance (Mulvey and Kim (2007)). Importantly, one can obtain better diversification benefits from industry-wide market segmentation, as compared to the current size/style break-outs. We discuss details of the issue in the later section.

The main objectives of this paper are as follows: 1) The Markowitz model requires estimating parameters for return distributions for the assets, and these are often hard to estimate. We employ momentum patterns to see if they help with a Markowitz model and several variants. 2) In addition, we compare the performance of several popular mean-variance models in various settings.

The remainder of the paper is organized as follows. In section II, we briefly discuss four asset allocation models employed in this paper – traditional Markowitz, Black-Litterman, Grauber-Hakansson, and robust optimization models. In the following section, we illustrate how the industry-wide equity market segmentation can provide better diversification benefits. Empirical results and conclusions follow.

II. Models: Markowitz Model and its Variants

In spite of its popularity, several issues arise regarding the practical implementation of the Markowitz model. A major issue is its sensitivity to input changes. Since the optimal portfolio from the mean-variance approach is chosen among the extreme points of the feasible region, small changes in the estimated parameters of the market distribution can lead radically different optimal points. As a consequence, relatively small errors in the parameter estimation can potentially cause a steep decrease of investment performance. Such a high sensitivity is undesirable for practical applications.

Many models have been proposed to overcome this shortcoming. A popular approach is to utilize robust estimators for the mean and the variance. Instead of using the unbiased estimators for the market distribution, one can reduce the estimation error by shrinking the sample mean and the sample covariance toward structured estimators. Such shrinkage methods are well documented in Jobson and Korkie (1981), Jorion (1986), Pastor (2000), and Resnick and Larsen (2001). Also, in a similar context, Black and Litterman (1990) propose a model to blend the investor's view with the market estimators. In their model, the investor's view, which is represented as a linear relation among the expected returns of the individual assets, is mixed to the market equilibrium via a Bayesian approach. See Satchell and Scowcroft (2000), and Idzorek (2004) for the detailed discussion.

An alternative approach is the portfolio re-sampling technique. In this approach, market parameters are re-sampled via Monte Carlo simulation, and the portfolio weights are obtained by averaging the optimal solutions of individual mean-variance problems with the generated estimators. The Michaud model (1998), for example, generates random samples from the estimated mean and variance, and obtains a new set of market parameter estimators from samples. The efficient frontier corresponding to this simulation is produced by minimizing a set of evenly spaced portfolio variances. After repeating the procedure sufficiently often, one can gain the re-sampled portfolio weight by averaging the optimal allocations with the same variance ranking. In a sense, this approach addresses the sensitivity issue by averaging the outputs from perturbed samples, while the robust estimator methods smoothes the estimators.

There also has been significant effort to improve the robustness of the optimal portfolio allocation by analytically reflecting the uncertainty of estimated parameters with the help of convex analysis. These models typically define the uncertainty set for the parameters and formulate the optimal allocation as a convex optimization problem to consider the worst case. Accordingly, this approach is referred to as a robust optimization model. For instance, when the mean return or the covariance is relaxed to lie in an ellipsoid, the mean-variance problem can be rewritten as a second order cone programming (SOCP) problem. If the covariance takes a finite number of matrices, it is formulated as a quadratic constrained quadratic programming (QCQP) problem. Note that both SOCP and QCQP are convex programs which can be solved efficiently. See Ben-Tal and Nemirovski (1995), El Ghaoui and Lebret (1997), Ben-Tal and Nemirovski (2001), and Boyd and Vandenberghe (2004) for the further discussion.

We pick one model for each of the approaches as well as the traditional Markowitz model to evaluate their historical performance. Black-Litterman model, Grauer-Hakansson model (1985), and SOCP model for the mean relaxation are chosen for the robust parameter estimation, re-sampled portfolio, and robust optimization approaches, respectably. Meucci (2005) and Fabozzi et al. (2007) discuss these models as well as other approaches in detail.

II.1. Robust Estimator: Black-Litterman

We employ a simplified version of Black-Litterman model described in Meucci (2007). With the normality assumption, let the prior on the market be μ and Σ. That is, the n-dimensional random return vector

$$r \sim N(\mu, \Sigma).$$

Also, let the view on the market can be expressed as the following linear function.

$$v = P\mu + \epsilon,$$

where P is k-by-n matrix corresponding to k views on the market along with k-vector v, and ϵ is the error term that follows $N(0, \Omega)$. Note that Ω represents the investor's confidence on the view. For simplicity, we set

$$\Omega = \left(\frac{1}{c} - 1\right) P\Sigma P^T.$$

For this specific choice of the uncertainty matrix Ω, c determines the confidence: values in Ω decrease as c increases from 0 to 1, which causes decrease in the variance for the view, thus making it more certain. For our tests, we use 0.01, 0.5 and 0.99 for the values of c. Then, it can be shown that the Black-Litterman estimators for the expected return and the covariance are given as follows.

$$\mu_{BL} = \mu + \Sigma P^T (P\Sigma P^T + \Omega)^{-1}(v - P\mu), \text{ and}$$

$$\Sigma_{BL} = \Sigma - \Sigma P^T (P\Sigma P^T + \Omega)^{-1} P\Sigma.$$

For the prior, μ and Σ, we employ the sample mean and the sample covariance of long-term (5 years) historical returns at each time period. P and v, which represent the investor's view on the market, are constructed to reflect the performance persistence of the recent winners, or the momentum effect. They are chosen in such a way that the average value of the expected returns of recent top 10% winners is higher than that of the remaining 90% by an arbitrary amount v in annualized return. In other words, for the index set I of the recent top 10% winners, and n_w and n_l, the number of winners and losers, respectably,

$$\frac{1}{n_w}\sum_{i\in I}\mu_i = \frac{1}{n_l}\sum_{j\notin I}\mu_j + v.$$

Therefore, P is 1-by-n vector, where

$$P_i = \frac{1}{n_w} \text{ for } i \in I, \text{and } P_j = \frac{1}{n_l} \text{ for } j \notin I.$$

We choose 3-, 6-, 9-, 12-, 24-, 36- and 60-month evaluation time lengths to obtain P and 1% for v. Once all input parameters are set, the mean-variance approach is employed to obtain the optimal portfolio allocation. Note that this approach is consistent with the long-only momentum strategies proposed in Mulvey and Kim (2007). They construct the long-only industry-level momentum portfolios by holding recent top 10% winner industries with equal weights in several stock markets. The portfolios have outperformed the benchmark market indices in most of the tested markets.

II.2. Re-sampled Portfolio: Grauer-Hakansson

Several authors have successfully implemented a sequence of single-period optimization model based on optimizing a Von Neumann-Morgenstern (VM) expected utility function. See, for example, Grauer and Hakansson (1985), and Mulvey et al. (2006). Also, Markowitz (1952) discusses the advantages of employing the model. Herein, we implement the model via an iso-elastic utility function:

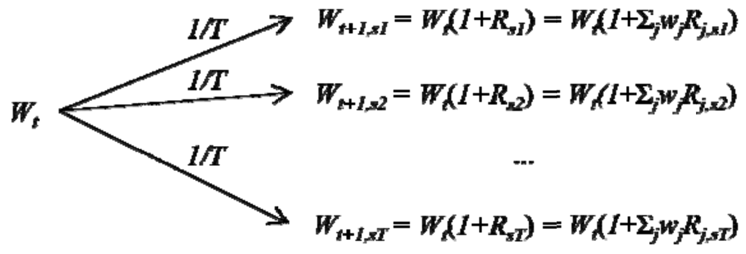
$$u(w) = \frac{1}{\gamma}w^{\gamma}, \qquad \text{where } \gamma \leq 1.$$

VM model is based on the one-step tree representation of scenarios as illustrated in Figure 1. Given the wealth W_t at time t, the future wealth at time $t+1$ for scenario s_i is $W_t(1+R_{s_i})$ with a probability of π_{s_i}, where R_{s_i} is the portfolio return between t and $t+1$. That is,

$$R_{s_i} = \Sigma_j w_j r_{j,s_i},$$

where w_j is the weight on asset j and r_{j,s_i} is the return of asset j for scenario s_i. We use each monthly observation as one scenario and set the probability equal across scenarios. For instance, when T-month historical data is employed for the scenario construction, the asset returns at month i is set to be the scenario return for scenario s_i, and $1/T$ is assigned as the probability for each scenario, thus having T scenarios total.

Figure 1: Tree representation of the Grauer-Hakansson model



The optimal portfolio weight is calculated from the following expected utility maximization problem with a non-negativity constraint on the portfolio weights, since this model does not utilize the mean-variance approach.

$$Max_{w\geq 0}\mathbb{E}U(W_{t+1}) = \frac{1}{\gamma}\sum_{i=1}^{T}\frac{W_{t+1,s_i}^{\gamma}}{T} = \frac{1}{\gamma T}\sum_{i=1}^{T}W_{t+1,s_i}^{\gamma} \quad \text{for } \gamma \leq 1$$

An efficient frontier is generated by varying the risk-aversion coefficient (γ); $\gamma = 1$ corresponds to the risk neutral case, and the resulting portfolio becomes more conservative as γ decreases. As in the Black-Litterman model, we adopt 3-, 6-, 9-, 12-, 24-, 36- and 60-month as the parameter estimation periods.

II.3. Robust Optimization: SOCP for Ellipsoidal Relaxation of μ

Consider the following mean-variance problem.

$$\begin{array}{ll} \text{Maximize} & w^T\mu \\ \text{Subject to} & w^T\Sigma w \leq \sigma_{target}^2 \\ & 1^T w = 1,\ w \geq 0 \end{array}$$

Suppose the parameter μ is uncertain, but is known to lie in an ellipsoid induced by $\bar{\mu}$ and P:

$$\mu \in \mathcal{E} := \{\bar{\mu} + Pu | d_2(u) \leq q\}, \text{where } d_2 \text{ is Euclidean norm.}$$

Note that the size of the ellipsoid increases as q gets large, so q can be interpreted as the degree of the uncertainty on μ.

Now suppose we strive to obtain the robust asset allocation in a sense that the solution is optimal under the worst-case scenario. Then the mean-variable problem can be restated as follows:

$$\begin{array}{ll} \text{Maximize} & \inf_{\mu\in\mathcal{E}} w^T\mu \\ \text{Subject to} & w^T\Sigma w \leq \sigma_{target}^2 \\ & 1^T w = 1,\ w \geq 0 \end{array}$$

Since for every given w,

$$\inf_{\mu\in\mathcal{E}} w^T\mu = w^T\bar{\mu} + \inf\{Pu|d_2(u) \leq q\} = w^T\bar{\mu} - qd_2(P^Tw) = w^T\bar{\mu} - q\sqrt{w^TPP^Tw}\,,$$

we have

$$\begin{array}{ll} \text{Maximize} & w^T\bar{\mu} - q\sqrt{w^TPP^Tw} \\ \text{Subject to} & w^T\Sigma w \leq \sigma_{target}^2\ . \\ & 1^T w = 1,\ w \geq 0 \end{array}$$

Therefore, the original problem with the ellipsoid relaxation can be expressed as

$$\text{Maximize} \quad w^T\bar{\mu} - z$$

$$\text{Subject to} \quad d_2(\Sigma^{\frac{1}{2}} w) \leq \sigma_{target}$$
$$1^T w = 1, \; w \geq 0$$
$$q d_2(P^T w) \leq z,$$

where $\Sigma^{1/2}$ is a Cholesky decomposition of Σ. The final form is SOCP, which can be solved relatively easily via convex optimization techniques. Note that the additional constraint ($qd_2(P^T w) \leq z$) plays a role of keeping w from moving toward the direction to which the uncertainty increases. See Boyd and Vandenberghe (2004), and Meucci (2007).

There are three input parameters that should be estimated for the implementation: $\bar{\mu}, \Sigma$ and P. We employ the sample mean and the sample covariance for $\bar{\mu}, \Sigma$. Also, for simplicity, we choose P in such a way that $PP^T = diag(\Sigma)$. In addition we vary q from 0 to 1 to investigate the effect of the uncertainty level. As in the previous cases, parameters are estimated from 3-, 6-, 9-, 12-, 24-, 36- and 60-month historical data, in order to evaluate the impact of various momentum-based rules.

II.4. Markowitz Model and General Experiment Settings

We adopt the following traditional mean-variance problem with the non-negativity constraint as the benchmark model.

$$\text{Maximize} \quad w^T \mu$$
$$\text{Subject to} \quad w^T \Sigma w \leq \sigma_{target}^2$$
$$1^T w = 1, \; w \geq 0$$

The sample mean and the sample covariance from 3-, 6-, 9-, 12-, 24-, 36- and 60-month historical data are employed for μ and Σ.

For all four models, we conduct the following sequential portfolio allocation: at the beginning of the evaluation period, input parameters as described above, and the optimal allocation are determined. Then, the assets are held for 3-, 6- or 12-month based on the optimal weights and rebalanced every month to the initial weights (i.e. fixed mix portfolios). After the holding period has ended, another set of the allocation is conducted. The procedures are repeated until the end of the sample period. Table 1 summarizes four models and accompanied parameters.

Table 1: Summary for Asset Allocation Models

Scheme	Model	Market Estimators and Parameters
Robust Estimator	Black-Litterman	- μ: sample mean from 60-month data - Σ: sample covariance from 60-month data - v and P: winners from 3-, 6-, 9-, 12-, 24-, 36- and 60-month data - $\Omega = \left(\frac{1}{c} - 1\right) P\Sigma P^T$.: confidence on investors view. (c=0.01, 0.5, and 0.99)
Re-sampled Portfolio	Grauer-Hakansson	- r_{j,s_i}: monthly return on asset j from 3-, 6-, 9-, 12-, 24-, 36- and 60-month data
Robust Optimization	SOCP for ellipsoidal μ	- $\bar{\mu}$: sample mean from 3-, 6-, 9-, 12-, 24-, 36- and 60-month data - Σ: sample covariance from 3-, 6-, 9-, 12-, 24-, 36- and 60-month data - P: diagonal elements of Σ
Markowitz Model	Mean-Variance Model	- μ: sample mean from 3-, 6-, 9-, 12-, 24-, 36- and 60-month data - Σ: sample covariance from 3-, 6-, 9-, 12-, 24-, 36- and 60-month data

III. Data: Benefits of Industry-wide Market Segmentation

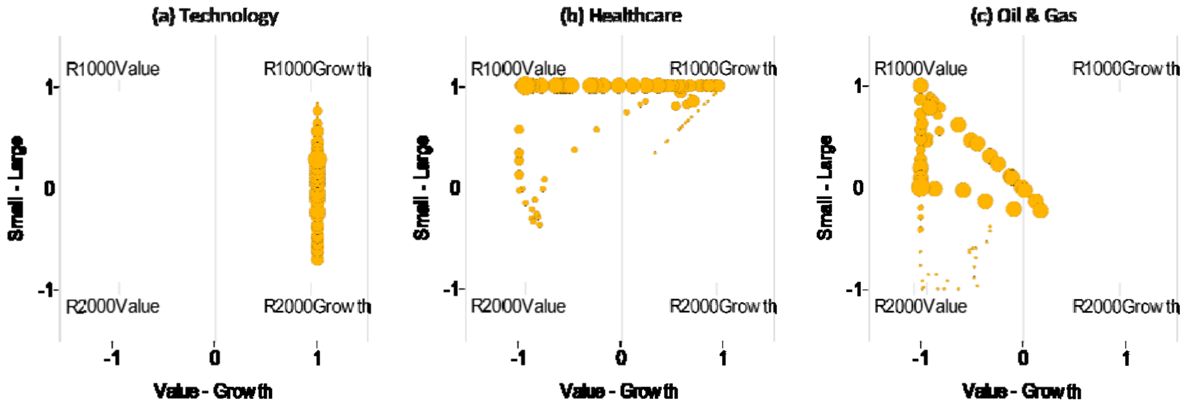
In this section, we discuss the importance of employing generic asset categories within an optimal portfolio model. Clearly, the performance of optimal asset allocation models is highly dependent upon the characteristics of the given assets. Efforts to find asset classes with good properties should precede the model selection. Current practical approaches typically divide the equity market based on the sizes and the style prospects of stocks (e.g., large-core, mid-growth, small-value, etc.). Since the size/style segmentation is simply a scheme to cut the group of investable vehicles, it is natural to ask if the current segmentation scheme can be improved.

In this regard, Mulvey and Kim (2008) illustrate that a segmentation scheme based on industry-level definitions can potentially improve performance of investment vehicles, as compared to the current size/style scheme. First, an industry-wide segmentation can provide more consistent membership over time. The reason is clear: firms cannot easily change industries which they belong to, while their sizes and growth perspectives vary. This property not only enables investors to track each of the segmentations easily but also potentially improve performance active funds; many mutual fund managers are typically bound to form their portfolios with stocks corresponding to the styles of the funds. Thus, fund managers may be forced to perform undesired portfolio adjustments to reflect the changes in the benchmark components, when the membership for each breakout changes. Such a procedure typically limits the fund managers' choices, which may lead to inferior investment performance.

Figure 2 shows how unstable the style/size classifications have been for the last decade as compared to the industry segmentation scheme. Here, we conduct style analysis over a recent time period in a sequential fashion. The technology industry (left in Figure 2) stays as growth-oriented, while its size has shrunken from large-cap to small-cap, and then grown back to large-cap. Similarly, the healthcare industry (center in Figure 2) has been classified as large-cap, while its growth perspective has changed over the sample period. Also, the oil and gas industry (right in Figure 2) has moved over the three quadrants of the style/size map. Since membership of firms within an industry is stable, it is clear that the constituents of style/size market break-outs have changed frequently.

Figure 2: Drift of Industries in Style/Size Map from December 1996 to November 2006

This figure illustrates drift of (a) technology, (b) healthcare, and (c) oil & gas industries in style/size map. The sample period is from December 1996 to November 2006. Relative position of each circle represents the size and the style of industries for each 24-month long period compared to 4 Russell indices on a rolling time basis. The circle sizes increase as time passes. The positions of the circles are based on regressions of industry returns against the Russell indices.

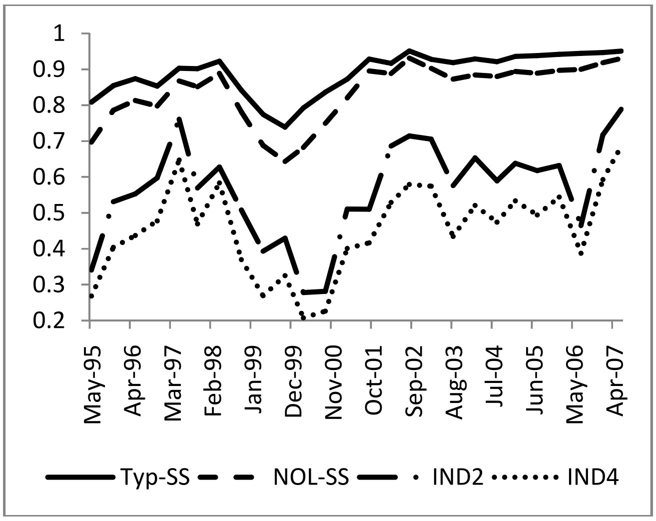


A critical benefit from the industry segmentation is improved diversification potential. Figure 3 depicts the average correlations of market breakouts from different segmentation schemes. Typ-SS and NON-SS represent the size/style segmentation scheme, while IND2 and IND4 correspond to the industry break-out. The message is clear: from the same stock market, one can obtain less correlated vehicles by applying the industry-wide segmentation scheme than the conventional size/style approach. It has a critical implication to the asset allocation models; it can provide better diversification for the broad asset allocation problems.

Figure 3: Average Correlations within Different Market Segmentation Schemes

This figure illustrates the average correlations for 4 different market breakouts defined in the top table. The sample period ranges from June 1995 to December 2007. For each of market segmentations, correlations for all possible index pairs are calculated from daily returns, and then averaged across those pairs. The unit time length is 6 months (126 trading days). See appendix for the list of Datastream sectors.

Description	Code	Indices Included
Typical Style/Size Breakouts	Typ-SS	R1000, R1000G, R1000V, RMid, RMidG, RMidV, R2000, R2000G, R2000V
Non-Overlapping Style/Size Breakouts	NOL-SS	R200G, R200V, RMidG, RMidV, R2000G, R2000V
DataStream Level 2 Sectors	IND2	10 Industries Indices
DataStream Level 4 Sectors	IND4	38 Industries Indices



IV. Test Results

In this section, the investment performance of the models introduced in section II is compared with several benchmark portfolios. To construct the portfolio corresponding to each model, we update estimators for the expected return and the covariance periodically, and employ optimization on a moving basis. There are two critical parameters: "look-back period" refers to the length of the historical data to estimate return and covariance matrices, and "holding period" shows the investment period for each asset allocation decision. For instance, at a given time point, a strategy with 3-month look-back period and 6-month holding period means that the inputs are estimated from the daily returns of past 3 months, and once the optimal allocation is obtained, the portfolio is held for 6 months. Note that all assets in a portfolio were rebalanced to their corresponding weights at the end of each month. Also, in order to eliminate the timing bias, the average returns from portfolio with different starting points are employed. The extra parameters for robust optimization and Black-Litterman have been chosen as mentioned in section II. In this context, we apply the Markowitz model and its variants to the industries as defined in Datastream. There are 38 industries with one market index, and we employ daily data from January 1976 to December 2007. Since the models require an initial period to estimate parameters, all constructed portfolios begin on January 1980. See the appendix for the list of industries.

IV.1. Model Comparisons: Which Model is better?

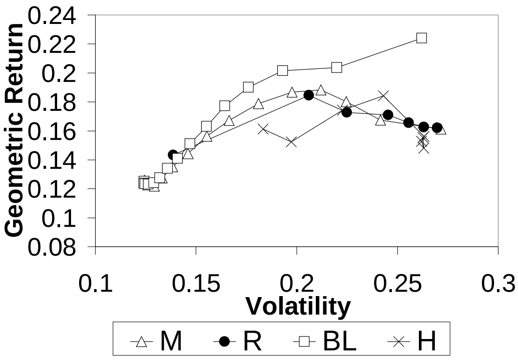
The historical performance is obtained after sequentially solving the relevant optimization problems, and calculating the return series for each model (Figure 4). Different risk tolerance levels are set to generate points along the line. For (a)-(f) in Figures 4, the holding period is fixed to 6 months, while the look-back period are set to 3-, 6-, 12-, 24-, 36- and 60-month, respectively.

When the look-back period is 3-month, the Black-Litterman model dominates the other models (Figure 4-(a)), particularly at the points corresponding to the highly risk portfolios. However, robust optimization and Grauber-Hakansson models produce better results when 12-months look-back period is adopted (From Figure 4-(c)). Furthermore, there is no dominating model in Figures 4-(e); returns are not significantly different from each other, while none of the approaches offer a promising decrease in volatility with respect to the others. In summary, for these tests, investment performance across different models is comparable to each other, and it highly depends on the parameter settings.

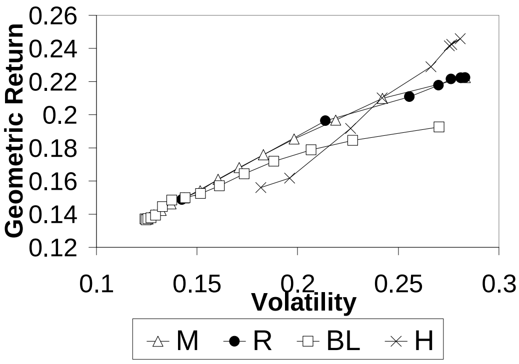
There are two other meaningful observations. First, the Markowitz model performs fairly well in practical settings over the entire sample period. Second, while the ex post performance lines in the mean-variance plane preserve upward slopes and concave shapes when the look-back period is equal or less than a year, they become downward sloping when it is longer than a year. Thus, performance deteriorates as the level of the risk tolerance increases for look-back period greater than one year. In fact, these findings coincide with the superior performance of the momentum strategy, and the discussion follows in the next subsection. Note that the results lead to the same conclusion when different holding periods are employed (3-, and 12-month), while the investment performance generally becomes worse as the holding period get longer.

Figure 4: Historical Performance of 4 Different Models from Jan.1980 to Dec.2007

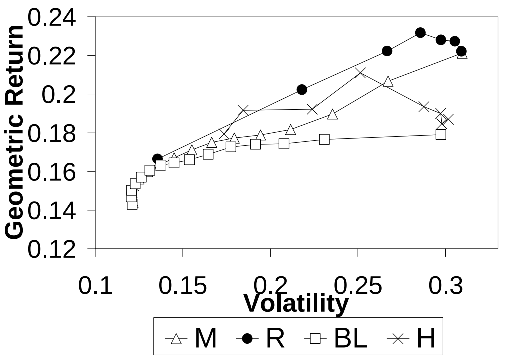
This figure illustrates the historical investment performance of the 4 different models – Markowitz (M), Black Litterman (BL), Grauber-Hakansson (H), and robust optimization (R) Models. The sample period is 1980 to 2007. Holding period is set to 6-month across all three figures, while look-back periods are set to (a) 3-month, (b) 6-month, (c) 12-month, (d) 24-month, (e) 36-month, and (f) 60-month.



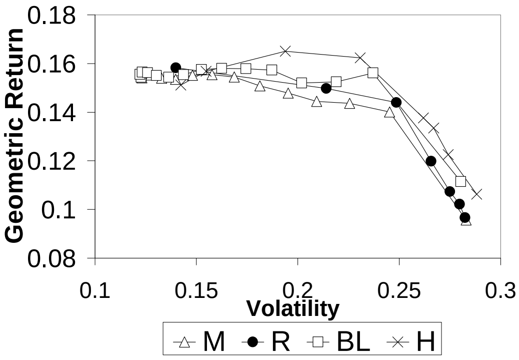
(a) 3-month look-back period



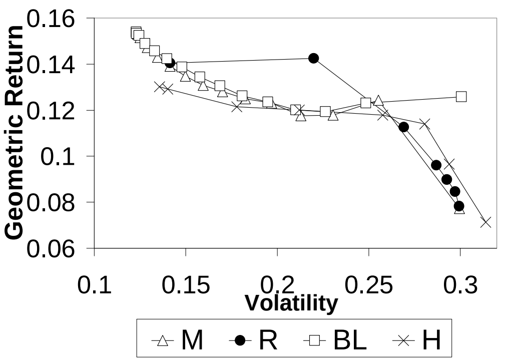
(b) 6-month look-back period



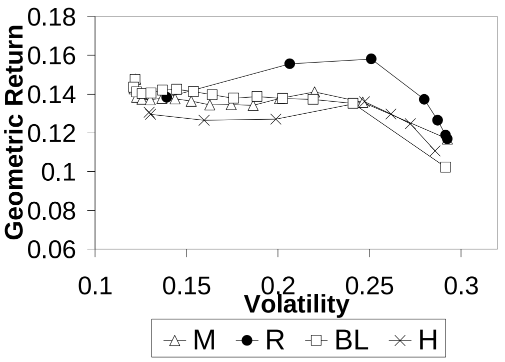
(c) 12-month look-back period



(d) 24-month look-back period



(e) 36-month look-back period



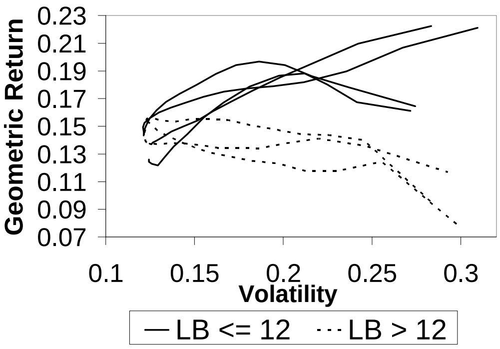
(f) 60-month look-back period

IV.2. Comparisons along Different Look-back Periods: Blending in Momentum Effects

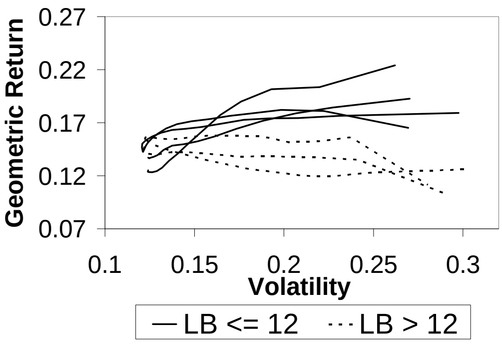
In this subsection, we evaluate several look-back periods. Figure 5 illustrates performance for different look-back periods (3-, 6-, 9-, 12-, 24-, 36-, 60-month) with a 6-month holding period. For all models, portfolios with short term look-back periods (3-, 6-, 9-, 12-month) show superior performance to ones with longer look-back periods (24-, 36-, and 60-month). Furthermore, the lines change shape from being upward sloping and concave to downward sloping and convex as the look-back period gets longer. As pointed out in the previous subsection, it implies that taking higher risk actually reduces the ex post returns. Tests with different holding periods (3-, and 12-month) yield similar results.

Figure 5: Performance across Different Look-back Periods

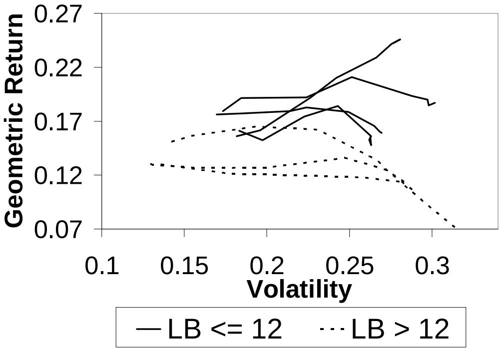
This figure illustrates the historical investment performance of each of the 4 different models with different look-back periods (LB). The sample period is 1980 to 2007. Holding period is set to 6 months across all three figures, while look-back periods are set to 3-, 6-, 9-, 12-, 24-, 36- and 60-month.



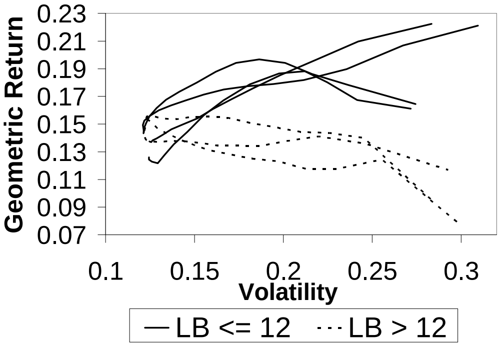
(a) Markowitz Model



(b) Black-Litterman



(c) Grauber-Hakansson



(d) robust optimization

These results imply that estimating market parameters from short look-back periods are better than ones from longer look-back periods. All models rely on the assumption that estimated returns and covariance matrices are proxies for the future values of these parameters. So, their performance depends on the persistence of these estimates along time. Since industries with better performance during the look-back periods would have higher weights in the portfolio for the subsequent period regardless of the model choice, it is evident that recent data provides better forecast on the distribution of the future returns.

Why is the shorter holding period better? The answers can be readily found from the equity price momentum effects: empirical studies suggest that winner stocks for the past 3 to 12 months outperform loser stocks for the following 3 to 12 months and show worse performance after 3 to 5 years. Therefore, when shorter holding periods are employed, the models put more weights on the recent winners, leading to a successful blend in of the momentum effects with optimal asset allocation models. In contrast, the portfolios from longer look-back period bet against the momentum effects, which would potentially cause inferior investment performance.

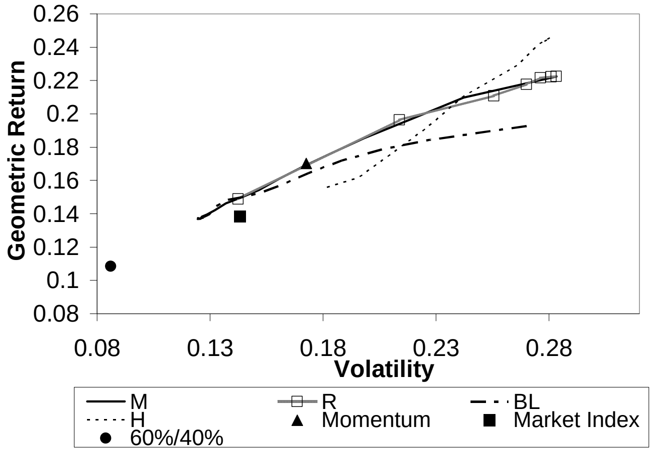
One significant question remains: is the momentum effect strong enough to improve investment performance of the asset allocation models? To see this, we compare the historical performance of the models to three benchmark portfolios – the market index, 60-40 fixed mix portfolio, and the long-only momentum portfolio (Figure 6). The 60-40 mix is constructed by investing 60% of the wealth to market index, and 40% to treasury bills with monthly rebalancing. Also, the performance of the long-only momentum portfolio is obtained by holding the winner industries based on the past 3-, 6-, 9-, and 12-month returns for 6 subsequent months. The chosen industries are equally weighted and rebalanced every month. For the optimal asset allocation portfolios, 6-month look-back period and 6-month holding period are employed; this setting provides relatively good investment performance for all four models.

Figure 6 provides a clear answer for the question. The performance of long-only momentum strategy lies on the best performance line of the optimal asset allocation models in the mean-variance plane for the entire sample period, meaning that the simple momentum rule has performed equivalently, if not better, to the optimization models. This implies that the momentum effects have been significant, and it can improve investment performance, if investors utilize the effects properly. It is interesting to note that the momentum strategy shows stronger performance during the second sub-period, which corresponds to the period after the momentum effects become popular due to the publication of Jegadeesh and Titman (1993).

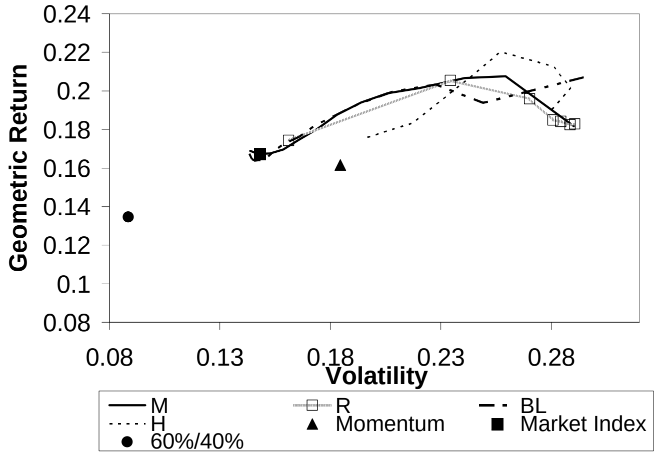
We are now ready to explain why the traditional Markowitz model has performed relatively well compared to its variants, especially when shorter holding periods are employed. The variants are designed to overcome the high sensitivity of the Markowitz model to input parameters. Therefore, in a sense, they smooth the result toward the direction that the optimal solutions wouldn't vary too much as the estimated input parameters change. So, higher weights on recent winners would be penalized, which would potentially discount the momentum effects. In contrast, the optimal weights from the Markowitz model is obtained via conventional optimization procedures, so it has a better chance of putting higher weights on the winners during the look-back period than its variants, thus provides better utilization of the momentum effects. It gives us a simple, yet effective recipe to exploit the momentum effects in the context of the optimal asset allocation: investment performance can be improved by estimating the market parameters from relatively recent historical data (3 to 12 months).

Figure 6: Performance Comparisons to Selected Benchmarks

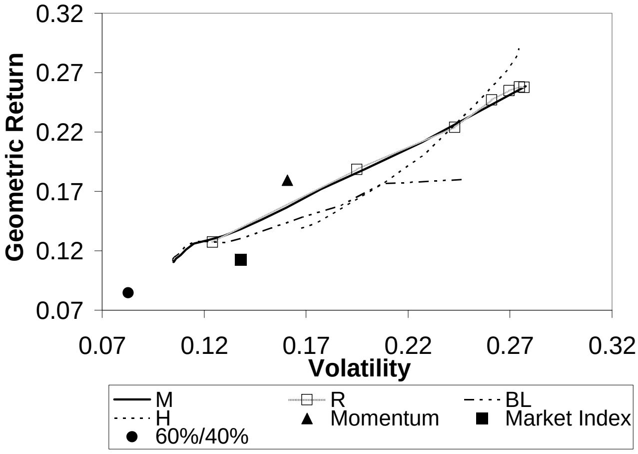
This figure illustrates the historical investment performance of 4 different models with 6-month look-back period and 6-month holding period. In addition, three benchmark portfolios are shown – market index, 60-40 fixed mix portfolio and long-only momentum strategy.



(a) Entire Sample Period: 1980-2007



(b) First Half of the Sample Period: 1980-1993



(c) Second Half of the Sample Period: 1994-2007

V. Conclusions and Future Directions

In this paper, we construct various portfolios from four different optimal asset allocation models, and compare the historical performance during 1980 to 2007. There are several meaningful findings: 1) the traditional Markowitz model has performed reasonably well as compared to its robust versions; 2) portfolios with shorter look-back periods (equal or less than a year) outperform ones with longer look-back periods in all cases; 3) these observations are in fact consistent with the momentum effects, which imply the recent winners tend to have better performance than the recent losers; and 4) investment performance can be improved by taking the momentum effects into account by utilizing the market parameter estimators from recent historical data.

What are possible extensions? First, it would be interesting to allow shorting of assets. Again, the momentum strategy could be the benchmark, since empirical results show that recent momentum losers will continue to underperform the market for subsequent periods. Shorting of industry-level assets is becoming more practical due to the emergence of the ETFs. In many cases, ETFs can be easily shorted.

Second, the described methodology can be applied to other extensions of the traditional Markowitz model, such as Stein estimators (Jorion (1986)). We suspect that the traditional Markowitz model will again perform relatively well.

The third domain for extending the analysis involves integrating asset management with borrowing (leverage) and other liability related issues – asset and liability management (ALM). In many of these applications such as pension plans, university endowments, and hedge funds, there are decided advantages to construct multi-period ALM models. The ex post success of momentum strategy shall apply to these optimization models. But this conjecture needs to be evaluated with real world experiences.

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Appendix

Datastream Industry Classification

Level 2 (10 Indices)	Level 3 (18 Indices)	Level 4 (38 Indices)
Oil & Gas	Oil & Gas	Oil & Gas Producers; Oil Equipment, Services & Distribution
Basic Materials	Chemicals	Chemicals
	Basic Resources	Forestry & Paper; Industrial Metals; Mining
Industrials	Construction & Materials	Construction & Materials
	Industrial Goods & Services	Aerospace & Defense; General Industrials; Electronic & Electrical Equipment; Industrial Engineering; Industrial Teleportation; Support Services
Consumer Goods	Automobiles & Parts	Automobiles & Parts
	Food & Beverage	Beverages; Food Producers
	Personal & Household Goods	Household Goods; Leisure Goods; Personal Goods; Tobacco
Health Care	Health Care	Health Care Equipment & Services; Pharmaceuticals & Biotechnology
Consumer Services	Retail	Food & Drug Retailers; General Retailers;
	Media	Media
	Travel & Leisure	Travel & Leisure
Telecommunication	Telecommunication	Fixed Line Telecommunication; Mobile Telecommunication
Utilities	Utilities	Electricity; Gas, Water & Multi-utilities
Financials	Banks	Banks
	Insurance	Nonlife Insurance; Life Insurance
	Financial Services	Real Estate; General Financials; Equity Investment Instruments
Technology	Technology	Software & Computer services; Technology Hardware & Equipment

Note: DataStream industry classification is almost identical to Dow-Jones/FTSE ICB (Industry Classification Benchmark).