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Modeling the dynamics of institutional, foreign, and individual investors through price consensus

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ABSTRACT

In this paper, we present a price consensus measure for understanding the dynamics among institutional, foreign, and individual investors. The proposed measure inversely estimates investors' daily views on the value of an asset, which incorporates the price consensus of the investor type. The price consensus measure is derived based on a rational expectation asset model and CARA utility function, and its effectiveness is empirically demonstrated by conducting cross-sectional analyses on historical trade data of the Korean stock market. These analyses demonstrate the advantage of using the price consensus measure when compared against modeling only net purchase amounts. Moreover, the findings show that institutional and foreign investors tend to have distinct long-term views while individual investors have views that are less extreme and thus showing characteristics of uninformed trades. Findings on short-term views exhibit information spillover from institutional and foreign investors to individuals.

1. Introduction

The dynamics and relation among the trades of institutional, foreign, and individual investors and a comparison among their performances have been topics of debate in academic research. For instance, Grinblatt, Titman, and Wermers (1995) and Wermers (1999) analyze characteristics of mutual funds, while Barber and Odean (2000) observe individual traders through household account data. Cohen, Gompers, and Vuolteenaho (2002) capture the reaction of individual and institutional investors to cash-flow news, and Choe, Kho, and Stulz (1999, 2005) study investment behavior of foreign investors in the Korean market. Moreover, there are also studies that compare the performance of various types of traders (Bae, Yamada, & Ito, 2006; Grinblatt & Keloharju, 2000; Karolyi, 2002; Kamesaka, Nofsinger, & Kawakita, 2003).

Most of the previous studies focus on net transaction amount for comparing the characteristics of each market player. However, to compare the market participants more inclusively, we believe that the price which they trade is also important. An investor could buy (sell) a stock not only because he or she evaluates the value of the stock to be high (low), but also because the stock price is temporarily lower (higher) than the investor's price consensus. For this reason, considering only transaction amounts could lead to uncomprehensive observations. Therefore, in this paper, we suggest a price consensus measure which incorporates stock price, stock volatility, and risk aversion of investors, in addition to net transaction amount.

The basic idea of the proposed measure is that if an investor purchases shares of a stock, it could be inferred that the estimated value of the stock by the investor is higher than the executed price. Furthermore, if an investor is more risk averse, the gap between the estimated value and executed price is expected to be larger. Through a price consensus measure, we are able to inversely approximate market participants' expected stock price. The measure allows considering not only net purchase amounts of market participants, but also the price that they trade at, when analyzing the dynamics of market participants.

A similar model was first proposed by Grossman and Stiglitz (1980), which is a noisy rational expectation model with a single asset where informed, uninformed, and noisy traders exist. The main contribution of our research is applying the model of Grossman and Stiglitz in order to incorporate price consensus and applying the theoretic model to analyze empirical data for providing more comprehensive implications on the behavior of different investors.[1] In contrast to their work, we assume all traders are informed traders with their own views on the value of a stock, while the quality of the views may differ. Thus,

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[1] The model of Grossman and Stiglitz (1980) could be extended to a multiple-asset case (Admati, 1985), but this paper focuses on the single-asset case to study the dynamics among institutional, foreign, and individual investors in a basic setting.

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1057-5219/

uninformed traders in Grossman and Stiglitz (1980) are also considered to be informed to a degree, who trade with disperse and uncertain information. In addition, daily closing prices are thought as the price that reach the equilibrium after intraday trades of institutional, foreign and individual investors based on their views. For the empirical analysis, we focus on investors in the Korean stock market since our proposed measure can be easily calculated for KOSPI (Korea Composite Stock Price Index) stocks from publicly available data of the Korea Exchange, such as data on historical stock prices and net purchase amounts of various investor types. We include analyses for 2008 and 2014, which can be considered as crash and normal market periods, respectively. Note that studying the behaviors of different types of investors is particularly important in emerging markets including Korea, because these markets are known to be vulnerable to the trades of foreign investors (see Choe et al., 1999, 2005). Nonetheless, our method can be also applied to other markets when net transaction data of different types of investors are available.

The remainder of the paper is organized as follows. The price consensus measure and its theoretic justification are presented in Section 2. In Section 3, empirical behavior of institutional, foreign, and individual investors is analyzed with the proposed price consensus measure. Section 4 concludes.

2. Price consensus measure

In this section, we present a measure that represents a price consensus of an investment group. We follow the rational expectations model of Grossman and Stiglitz (1980). We begin by introducing the market model in general form, and then explain how the model is extended for measuring price consensus, including a discussion on parameter estimation.

2.1. The general model

There are three types of agents in the market: institutional, foreign, and individual investors. The number of agents at time t in each type are denoted as N^t_{Ins}, N^t_{For} and N^t_{Ind}, respectively. All agents are risk averse and their utility function is defined as CARA (constant absolute risk aversion) utility function with positive risk aversion represented by γ_{Ins}, γ_{For}, and γ_{Ind} for each agent type, respectively. Agents invest in a risky asset, such as stocks, and a risk-free asset. The risk-free asset has a risk-free rate of return r_f, and the price of the risky asset, denoted as S, is composed of two parts,

$$S = \mu + \varepsilon$$

where μ is the value of the risky asset which is unknown to all agents, and ε represents noise. The noise term is normally distribution with a mean of zero and variance σ^2 while the variance is assumed to be known to all agents. Agents are assumed to have daily views or price consensus about μ and trades depend on the views. Their views may be formed based on cash flow information of the company or may reflect sentimental subjective views on the price of the risky asset. The price consensus on the value of the risky asset μ at time t for institutional, foreign, and individual investors are denoted as μ^t_{Ins}, μ^t_{For}, and μ^t_{Ind}, respectively. Furthermore, we assume the economy as an open system, and this assumption allows us to disregard the stock position of an agent prior to a specific trade.

At time t, an agent of type i will trade to maximize the expected utility,

$$\begin{aligned} &\max E\left[-exp^{-\gamma_i\left(d_i^t S+\left(W_i-d_i^t p^t\right)\left(1+r_f\right)\right)}\right] \\ &= \max -exp^{-\gamma_i\left(d_i^t E_i[S]+\left(W_i-d_i^t p^t\right)\left(1+r_f\right)\right)+\frac{1}{2}\gamma_i^2 {d_i^t}^2 Var(S)} \end{aligned}$$

where

d_i denotes the demand for the risky asset of the agent i, W_i is wealth of the agent i, and p^t is the price of the risky asset at time t. The optimal demand d^t_i of agent i at time t can be expressed as

$$d_i^t = \frac{E_i[S]-(1+r_f)p^t}{\gamma_i Var(S)} = \frac{\mu_i^t-(1+r_f)p^t}{\gamma_i\sigma^2} \tag{1}$$

where E_i is the conditional expectation based on the investors' price consensus. Thus, the demand of agent i at time t is proportional to the difference between the price consensus of the risky asset and the total return of investing the market value of the risky asset at the risk-free rate. On the other hand, the demand of the agent is reciprocal to the risk aversion and the risk level of the risky asset. The equilibrium price of the risky asset is settled to satisfy the market clearing condition: market clears when the demands of buying and selling offset each other,

$$\sum_i D_i^t = \sum_i N_i d_i^t = \sum_i N_i \frac{\mu_i^t-(1+r_f)p^t}{\gamma_i\sigma^2} = 0 \tag{2}$$

where D^t_i represents the total demand of all investors of agent type i. By rearranging the formulation given by Eq. (2), we can show that price is settled as a discounted convex combination of agents' belief on the price as follows

$$p^t = \frac{1}{1+r_f}\left(\frac{\frac{N_{Ins}}{\gamma_{Ins}}}{\frac{N_{Ins}}{\gamma_{Ins}}+\frac{N_{For}}{\gamma_{For}}+\frac{N_{Ind}}{\gamma_{Ind}}}\mu_{Ins}^t+\frac{\frac{N_{For}}{\gamma_{For}}}{\frac{N_{Ins}}{\gamma_{Ins}}+\frac{N_{For}}{\gamma_{For}}+\frac{N_{Ind}}{\gamma_{Ind}}}\mu_{For}^t+\frac{\frac{N_{Ind}}{\gamma_{Ind}}}{\frac{N_{Ins}}{\gamma_{Ins}}+\frac{N_{For}}{\gamma_{For}}+\frac{N_{Ind}}{\gamma_{Ind}}}\mu_{Ind}^t\right). \tag{3}$$

In addition, it can be noticed that larger the number of agents and lower the risk aversion, the price impact of the belief becomes stronger. In Section 3.2, with price and demand data, we show how to decompose price into information of three types of agents.

2.2. Parameter estimation from data

With some assumptions, the general formulation explained in the previous section can be modeled using data that only contains demand of each agent, represented by net purchase amounts and price of the risky asset. The variance of the risky asset denoted as σ^2 that is known to all market players is estimated as the sample variance of price of the risky asset. Moreover, the distribution of μ^t_i is modeled as an independent normal distribution $N(\theta_i, s^2_i)$ with a fixed variance s^2_i, and the daily risk-free rate is set to zero.

With these assumptions, the likelihood function of the model can be written as

$$\begin{aligned} \text{Likelihood} &= \prod_i\prod_t N\left(D_i|N_i\frac{\theta_i-p^t}{\gamma_i\sigma^2}, N_i^2\frac{s_i^2}{\gamma_i^2\sigma^4}\right) \\ &= \prod_i\prod_t N\left(D_i|\frac{\theta_i-p^t}{\hat{\gamma}_i\sigma^2}, \frac{s_i^2}{\hat{\gamma}_i^2\sigma^4}\right) \end{aligned} \tag{4}$$

where $\hat{\gamma}_i$ substitutes γ_i/N_i for simplicity. Our aim is to find θ_i and $\hat{\gamma}_i$ that maximize the likelihood function. The following proposition illustrates how the log of the likelihood function can be maximized.

Proposition 1. *For agent i*, based on the likelihood function given by Eq. (4)

(a) the optimal θ_i that maximizes the likelihood function given $\hat{\gamma}_i$ is

$$\theta_i = \overline{p} + \overline{D}_i\hat{\gamma}_i\sigma^2,$$

Table 1
Inferred parameters of investors invested in Samsung Electronics in 2014 1Q.

	Institutional investor	Foreign investor	Individual investor
$\hat{\gamma}_i$	9.169e − 10	1.098e − 09	9.862e − 10
θ_i	1,288,891	1,302,475	1,300,609

(b) the optimal $\hat{\gamma}_i$ that maximizes the likelihood function is

$$\hat{\gamma}_i = \frac{-\sum_t (D_i^t - \overline{D}_i)(p^t - \overline{p}) + \sqrt{\left(\sum_t (D_i^t - \overline{D}_i)(p^t - \overline{p})\right)^2 + 4Ts_i^2 \sum_t (D_i^t - \overline{D}_i)^2}}{2\sigma^2 \sum_t (D_i^t - \overline{D}_i)^2},$$

(c) based on (b) and the optimal demand given by (1), the price consensus μ_i^t becomes

$$\mu_i^t = p^t + \hat{\gamma}_i \sigma^2 D_i^t$$

where $\overline{D}_i$ and $\overline{p}$ are mean values of D_i^t and p^t during the period, respectively, and T is the total time period.

Proof. See Appendix A

The equation in Proposition 1(c) can be interpreted as the sum of two components, where the first component is the price of the risky asset, and the second component is the net purchase amount of agent i multiplied by a measure of risk price, represented by $\hat{\gamma}_i \sigma^2$, that the agents takes for each transaction amount. In addition, by Eq. (3), the stock price can be decomposed into a convex combination of three agents' price consensus as

$$\frac{\frac{1}{\hat{\gamma}_{Ins}}}{\frac{1}{\hat{\gamma}_{Ins}} + \frac{1}{\hat{\gamma}_{For}} + \frac{1}{\hat{\gamma}_{Ind}}}\mu_{Ins}^t + \frac{\frac{1}{\hat{\gamma}_{For}}}{\frac{1}{\hat{\gamma}_{Ins}} + \frac{1}{\hat{\gamma}_{For}} + \frac{1}{\hat{\gamma}_{Ind}}}\mu_{For}^t + \frac{\frac{1}{\hat{\gamma}_{Ind}}}{\frac{1}{\hat{\gamma}_{Ins}} + \frac{1}{\hat{\gamma}_{For}} + \frac{1}{\hat{\gamma}_{Ind}}}\mu_{Ind}^t$$
$$= p^t + \frac{\sigma^2}{\frac{1}{\hat{\gamma}_{Ins}} + \frac{1}{\hat{\gamma}_{For}} + \frac{1}{\hat{\gamma}_{Ind}}}\left(D_{Ins}^t + D_{For}^t + D_{Ind}^t\right) = p^t.$$

2.3. An example on Samsung electronics

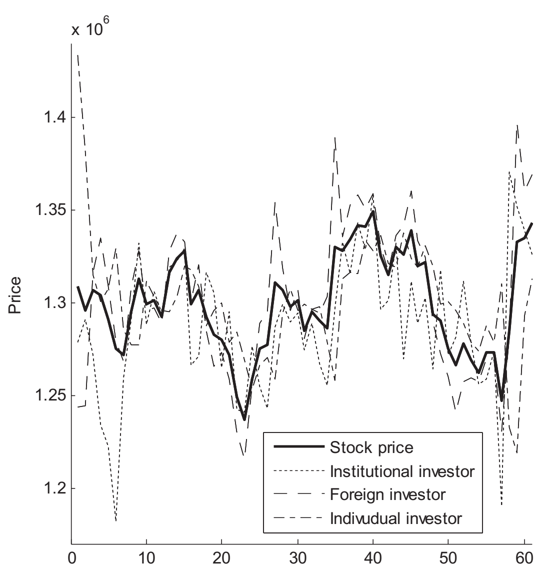
Before performing a detailed empirical analysis in the next section, we demonstrate an example on investors of Samsung Electronics. The daily close price of Samsung Electronics stock and the daily net purchase amounts of institutional, foreign, and individual investors in the stock in the first quarter of 2014 are used. The stock variance and the variance of each agent's consensus are estimated as the variance of stock price during the period, and this allows us to make inferences on $\hat{\gamma}_i$, θ_i, and μ_i^t.

As shown in Table 1, in the first quarter of 2014, foreign investors overestimated the value of Samsung Electronics on average while institutional investors underestimated the value on average. Furthermore, foreign investors have the largest $\hat{\gamma}_i$ whereas institutional investors have the smallest $\hat{\gamma}_i$, where a large $\hat{\gamma}_i$ means that there is a fewer number of investors for that investor type or that the investors are more risk averse. The values of $\hat{\gamma}_i$, which are calculated as the risk aversion divided by the number of the investors, are estimated as about 1e-9, where a reasonable magnitude of estimated risk aversion parameters are shown to be in the range of 1 to 10 in the economics literature.

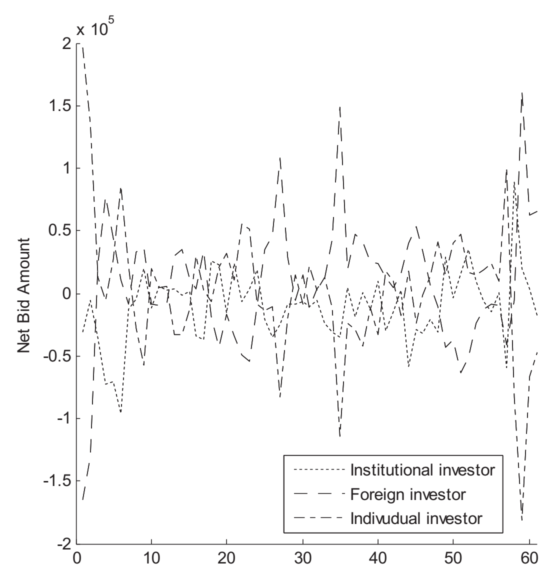
In Fig. 1, the daily time series of price consensus and net purchase amount of the three investor types are plotted. In Panel A of Fig. 1, daily price consensuses of three investors are plotted, which allows us to infer the price that the investors thought were reasonable for stocks. The advantage of our approach is evident from Panel B of Fig. 1, since it is difficult to gain information on whether the investors overestimate or underestimate stock prices when only observing net purchase amounts.

3. Empirical analysis

In this section, further empirical analyses on the Korean market are performed with emphasis on the measure suggested in Section 2. We



A. Stock price and price consensus



B. Net purchase amounts

Fig. 1. Price consensus and net purchase amounts in 2014 1Q.

Table 2
Summary of order statistics of the 388 samples in 2014.

	θ_i (mean of price consensus)				$\hat{\gamma}_i$ (risk aversion over N_i)			
	Institutional investor	Foreign investor	Individual investor	Sum	Institutional investor	Foreign investor	Individual investor	Sum
Max	138	135	115	388	58	157	173	388
Mid	115	120	153	388	119	100	169	388
Min	135	133	120	388	211	131	46	388
Sum	388	388	388		388	388	388	

Table 3
Summary of order statistics of the 388 samples in 2008.

	θ_i (mean of price consensus)				$\hat{\gamma}_i$ (risk aversion over N_i)			
	Institutional investor	Foreign investor	Individual investor	Sum	Institutional investor	Foreign investor	Individual investor	Sum
Max	179	69	96	344	125	107	137	344
Mid	109	97	138	344	119	80	145	344
Min	56	178	110	344	100	157	62	344
Sum	344	344	344		344	344	344	

continue to focus on the relation among the three types of investors: institutional, foreign, and individual investors. Our analysis consists of one long term quarterly analysis based on the estimated parameters and two short term daily analyses with correlation test and vector autoregression.

3.1. Data and methods

For the empirical study, we focus on two one-year periods, 2008 and 2014, in order to analyze behavior during normal as well as abnormal periods, where 2008 is considered as a crash period and 2014 is considered as a relatively stable period. The constituent securities of KOSPI 100 index on December 30, 2008 and December 30, 2014 are used for investigating investors 2008 and 2014, respectively. This leads to analyzing 86 stocks of year 2008 and 97 stocks of year 2014, which were traded throughout the entire year. Public data from the Korea Exchange were collected, which include daily close price and net purchase amounts of institutional, foreign, and individual investors.[2]

The distributions of price consensus and the number of investors are assumed to be independent for different stocks in order to focus on the dynamics of different investor types. The risk aversion over the number of investors $\hat{\gamma}_i$ and the mean of consensus distribution θ_i are newly calculated for each quarter to take account of the long term inconsistency of the parameters. The quarterly estimation period of the parameters is based on the fact that companies publish quarterly reports. The stock variance σ^2, which is known to agents, and the variance of the consensus signal s_i^2 are estimated as the stock price variance during the quarter as discussed earlier.

With the estimated values for $\hat{\gamma}_i$, θ_i, and also the price consensus time series μ_i^t, we next demonstrate several analyses on cross-sectional behavior. Here, θ_i could be interpreted as relatively long-term price consensus of the agents, and the perturbation added to θ_i, which is the daily price consensus time series μ_i^t, represents short-term news or information. The distinction between long-term and short-term information becomes important in our analyses.

3.2. Long term analysis: mean of price consensus and risk aversion

Observations of the 97 stocks in 2014 over the four quarters provide 388 sampled values for θ_i and $\hat{\gamma}_i$. Similarly, 86 stocks over the four quarters in 2008 provide 344 sampled values. Table 2 summarizes the order statistics of θ_i and $\hat{\gamma}_i$ among the three types of investors in 2014; the occurrences of maximum, minimum, and middle values for the 388 samples are shown. Likewise, Table 3 presents a summary for 2008.

Tables 2 and 3 reveal some interesting patterns on the ranking among the three investor types. First, the mean of price consensus for institutional and foreign investors tend to be polarized to the maximum or minimum among the three investors in both periods. However, in case of individual investors, their mean of price consensus tends to stay in between the other two investor types. These results imply that institutional and foreign investors have unique subjective price consensuses, but that is not the case for individual investors, which implies that individual investors are the most uninformed traders. Specifically, during the crash period of 2008, institutional investors are more polarized to overestimate the stock value and foreign investors are more polarized to underestimate the value, while they evenly overestimate and underestimate the values during the normal period of 2014.

Second, Tables 2 and 3 also show that the three investor types have different rank trends on $\hat{\gamma}_i$. In 2014, institutional investors tend to have minimum $\hat{\gamma}_i$ among the three types of investors, individual investors tend to have maximum or middle level of $\hat{\gamma}_i$, and foreign investors tend to stay between institutional and individual investors. As briefly mentioned when discussing the example on Samsung Electronics, high $\hat{\gamma}_i$ value could be resulted from various reasons. As $\hat{\gamma}_i$ is computed as γ_i/N_i, a high $\hat{\gamma}_i$ can be caused by a high γ_i or a small number of investors for type i. In addition, due to the use of net purchase amount for representing demand, it should be noted that a small net transaction amount can be caused not by a high risk aversion but by inconsistency in trade direction among investors of the same type, in which case transactions will be cancelled out when computing the net transaction amount.

In Eq. (3), since the price model is set as a convex combination of three price consensuses with the coefficients proportional to the inverse of $\hat{\gamma}_i$, the investor type that has lower $\hat{\gamma}_i$ can be inferred as having more severe price impact according to their consensus level. To summarize, the results for institutional investors in Table 2 can be interpreted as three possible situations: institutional investors are more likely to be risk-seeking investors, the number of institutional traders are large in the market, or institutional traders make similar decisions (i.e., herding). In comparison, individual investors can be inferred as being more risk-averse, having a small number of traders, or making diverse decisions among themselves. Furthermore, institutional investors cause stronger price impacts on the market.

On the other hand, the risk aversion of investors during the crash period of 2008 reveal another interesting behavior; institutional and individual investors tend to have maximum or middle level of $\hat{\gamma}_i$ and foreign investors tend to have minimum $\hat{\gamma}_i$. It implies that foreign investors

[2] Data available at www.krx.co.kr.

Table 4
Coefficients of correlations, autocorrelations, and cross-correlations in 2014.

Panel A. Correlation tests on investors' price consensuses and stock returns

	Institution's price consensus						Foreigner's price consensus					
Lag (period)	5	4	3	2	1	0	5	4	3	2	1	0
Inst. price consensus	0.29***	0.32***	0.36***	0.43***	0.57***	1.00***	0.21***	0.23***	0.26***	0.30***	0.35***	0.36***
For. price consensus	0.32***	0.34***	0.36***	0.39***	0.36***	0.36***	0.33***	0.38***	0.43***	0.49***	0.55***	1.00***
Ind. price consensus	0.41***	0.44***	0.46***	0.47***	0.46***	0.06***	0.41***	0.44***	0.47***	0.49***	0.49***	0.18***
Stock return	−0.08***	−0.10***	−0.11***	−0.12***	−0.13***	0.41***	−0.10***	−0.11***	−0.12***	−0.14***	−0.17***	0.31***

Panel B. Correlation tests on investors' net transaction amounts and stock returns

	Institution's net transaction						Foreigner's net transaction					
Lag (period)	5	4	3	2	1	0	5	4	3	2	1	0
Inst. net transaction	0.06***	0.08***	0.12***	0.17***	0.33***	1.00***	−0.08***	−0.09***	−0.11***	−0.12***	−0.16***	−0.42***
For. net transaction	−0.02**	−0.05***	−0.07***	−0.11***	−0.23***	−0.42***	0.07***	0.10***	0.13***	0.17***	0.26***	1.00***
Ind. net transaction	−0.04***	−0.03***	−0.05***	−0.07***	−0.13***	−0.58***	0.01	0	0	−0.02***	−0.06***	−0.40***
Stock return	0	−0.01	−0.02**	−0.01*	0.02**	0.43***	−0.03***	−0.02***	−0.03***	−0.03***	−0.05***	0.28***

*** Significant at 0.1%.
** Significant at 1%.
* Significant at 5%.

Table 4 (*continued*)

Panel A. Correlation tests on investors' price consensuses and stock returns

	Individual's price consensus						Stock return					
Lag (period)	5	4	3	2	1	0	5	4	3	2	1	0
Inst. price consensus	0.15***	0.16***	0.18***	0.19***	0.18***	0.06***	0.08***	0.08***	0.09***	0.13***	0.22***	0.41***
For. price consensus	0.19***	0.22***	0.25***	0.28***	0.32***	0.18***	0.10***	0.09***	0.09***	0.10***	0.04***	0.31***
Ind. price consensus	0.33***	0.35***	0.40***	0.44***	0.52***	1.00***	0.06***	0.08***	0.06***	0.05***	0.02***	−0.50***
Stock return	−0.07***	−0.07***	−0.08***	−0.09***	−0.13***	−0.50***	−0.01	−0.02***	−0.03***	−0.04***	−0.05***	1.00***

Panel B. Correlation tests on investors' net transaction amounts and stock returns

	Individual's net transaction						Stock return					
Lag (period)	5	4	3	2	1	0	5	4	3	2	1	0
Inst. net transaction	0.01	0	−0.02**	−0.06***	−0.17***	−0.58***	0	0	0.02**	0.05***	0.17***	0.43***
For. net transaction	−0.05***	−0.04***	−0.04***	−0.04***	0.01*	−0.40***	0.03***	0.02**	0.02**	0.02**	−0.09***	0.28***
Ind. net transaction	0.03***	0.03***	0.06***	0.10***	0.19***	1.00***	−0.03***	−0.01	−0.04***	−0.07***	−0.11***	−0.72***
Stock return	0.03***	0.04***	0.04***	0.05***	0.03***	−0.72***	−0.01	−0.02***	−0.03***	−0.04***	−0.05***	1.00***

during crash periods act like institutional investors during normal periods. Comparing the results of 2008 and 2014 shows that foreign investors underestimated the stocks in KOSPI 100 index and show the lowest risk aversion in financial crisis periods. This finding may explain the rapid exodus of foreign investors from the Korean market in 2008.

3.3. Short term analysis: correlation test

In this section, correlation tests on the price consensus and daily stock returns are performed to examine the relations between stock returns and the price consensus of the three investor types. This test will provide implications on the relation of short-term information of the three types of agents and stock returns. Furthermore, correlation between net purchase amounts and stock returns are also collected to compare the analytic power of the measure. Correlation tests that are performed include correlations, autocorrelations, and cross-correlations up to five lags, and the results in 2014 are contained in Table 4 and the results in 2008 are shown in Table 5. In Tables 4 and 5, Panel A summarizes the results for price consensus, and Panel B shows the results for net transaction amount.

We focus on the relationship with stock return and not stock price because stock price and investors' consensus are dependent to one another as shown in Eq. (3) by the definition of price consensus. In both panels, the columns indicated with a lag of zero list correlations; these values are shown in dark gray in Tables 4 and 5. For results with a lag greater than zero, autocorrelation is computed when the row and column labels are identical; these values are shown in light gray in the tables. The remaining values in both tables (i.e., without gray shading) contain cross-correlation with having the indicated number of lags on the column label series.

In Panel A of Table 4, institutional, foreign, and individual investors have positive autocorrelations above 0.3 and one lag autocorrelations above 0.5. Moreover, the daily price consensuses of the three agents are all positively correlated. Institutional and foreign investors are more correlated at 0.36 compared to correlations between individual and institutional investors as well as individual to foreign investors, which are 0.06 and 0.18, respectively. This implies that individual investors behave as contrarian in the short-term, or that they are isolated from the daily short-term information that reaches institution and foreign investors. Stock returns are positively correlated with the price consensuses of institution and foreign investors on the same day, but negatively correlated with the consensus of individual investors. Also, stock returns have all negative cross-correlations with past price consensuses, and all positive cross-correlations with future price consensuses. This shows that all investors tend to overestimate their price consensus to the past winners, similar to the case of institutional investors in Grinblatt et al. (1995) and foreign investors in Grinblatt and Keloharju (2000). However, stock returns are negatively correlated with the past consensus changes though it is positively correlated with the present price consensus changes of institutions and foreigners, which implies the short-term overshoot of the price consensus changes of investors. Furthermore, we can see that individual investors have relatively higher positive correlation with other investors' past price consensus, and this could be interpreted as short-term information spillover from institutional and foreign investors, which is also observed in the vector auto-regression test results presented in the following section.

In order to compare our proposed price consensus measure with only using net transaction amounts, Panel B of Table 4 presents findings of correlation tests with net buying quantity. The values are less significant than in Panel A and, thus, shows the effectiveness of our price consensus model. Moreover, the autocorrelation values are positive as in Panel A, but the values for correlation and cross-correlations are negative in Panel B. This is because of the dependency structure of the net purchase amounts of institutional, foreign, and individual investors, which sum to zero. This dependency structure interrupts from analyzing the relation of market players.

Table 5 presents the same type of results as in Table 4, but for 2008, which is considered a crisis period. The findings during normal and crash periods show overall similar patterns except few minor differences. Autocorrelations of all investors' price consensus and cross-correlations between the investors decrease in 2008, which means more volatile and inconsistent views are made in the crisis period. Similar to Table 4, Panel B of Table 5 also shows less significant results than Panel A, which demonstrates the limit of only focusing on net transaction amounts.

3.4. Short term analysis: daily vector auto-regression

The findings from the previous section are further analyzed using auto-regression models. Similar to the analysis of Griffin, Harris, and Topaloglu (2003), we perform two daily vector auto-regressive (VAR) test with five lags with 97 securities from January 2, 2014 to December 30, 2014 and 86 securities from January 2, 2008 to December 30, 2008. A VAR test enables analyzing the cause-and-effect relationship that cannot be measured with correlation tests. The first VAR test is for the relationship between the daily price consensus of each market player at t, which are denoted as μ^t_{Ins}, μ^t_{For}, and μ^t_{Ind}, and the stock return at t, expressed as R^t. The system of equations become

$$\begin{aligned}
\mu^t_{Ins} &= \alpha + \sum_{i=1}^{5}\beta_i^{Ins}\mu^{t-i}_{Ins} + \sum_{i=1}^{5}\beta_i^{For}\mu^{t-i}_{For} + \sum_{i=1}^{5}\beta_i^{Ind}\mu^{t-i}_{Ind} + \sum_{i=1}^{5}\lambda_i R^{t-i} + \delta_t^{Ins}\\
\mu^t_{For} &= \alpha + \sum_{i=1}^{5}\beta_i^{Ins}\mu^{t-i}_{Ins} + \sum_{i=1}^{5}\beta_i^{For}\mu^{t-i}_{For} + \sum_{i=1}^{5}\beta_i^{Ind}\mu^{t-i}_{Ind} + \sum_{i=1}^{5}\lambda_i R^{t-i} + \delta_t^{For}\\
\mu^t_{Ind} &= \alpha + \sum_{i=1}^{5}\beta_i^{Ins}\mu^{t-i}_{Ins} + \sum_{i=1}^{5}\beta_i^{For}\mu^{t-i}_{For} + \sum_{i=1}^{5}\beta_i^{Ind}\mu^{t-i}_{Ind} + \sum_{i=1}^{5}\lambda_i R^{t-i} + \delta_t^{Ind}\\
R^t &= \alpha + \sum_{i=1}^{5}\beta_i^{Ins}\mu^{t-i}_{Ins} + \sum_{i=1}^{5}\beta_i^{For}\mu^{t-i}_{For} + \sum_{i=1}^{5}\beta_i^{Ind}\mu^{t-i}_{Ind} + \sum_{i=1}^{5}\lambda_i R^{t-i} + \delta_t^{R}
\end{aligned}$$

where μ^t_{Ins}, μ^t_{For}, μ^t_{Ind}, and R^t are standardized to its series.

In addition, similar VAR tests with net purchase amount of investors at t, denoted as D^t_{Ins}, D^t_{For}, and D^t_{Ind}, instead of price consensus are performed for comparison. In this case, equations become

$$\begin{aligned}
D^t_{Ins} &= \alpha + \sum_{i=1}^{5}\beta_i^{Ins}D^{t-i}_{Ins} + \sum_{i=1}^{5}\beta_i^{For}D^{t-i}_{For} + \sum_{i=1}^{5}\beta_i^{Ind}D^{t-i}_{Ind} + \sum_{i=1}^{5}\lambda_i R^{t-i} + \delta_t^{Ins}\\
D^t_{For} &= \alpha + \sum_{i=1}^{5}\beta_i^{Ins}D^{t-i}_{Ins} + \sum_{i=1}^{5}\beta_i^{For}D^{t-i}_{For} + \sum_{i=1}^{5}\beta_i^{Ind}D^{t-i}_{Ind} + \sum_{i=1}^{5}\lambda_i R^{t-i} + \delta_t^{For}\\
D^t_{Ind} &= \alpha + \sum_{i=1}^{5}\beta_i^{Ins}D^{t-i}_{Ins} + \sum_{i=1}^{5}\beta_i^{For}D^{t-i}_{For} + \sum_{i=1}^{5}\beta_i^{Ind}D^{t-i}_{Ind} + \sum_{i=1}^{5}\lambda_i R^{t-i} + \delta_t^{Ind}\\
R^t &= \alpha + \sum_{i=1}^{5}\beta_i^{Ins}D^{t-i}_{Ins} + \sum_{i=1}^{5}\beta_i^{For}D^{t-i}_{For} + \sum_{i=1}^{5}\beta_i^{Ind}D^{t-i}_{Ind} + \sum_{i=1}^{5}\lambda_i R^{t-i} + \delta_t^{R}
\end{aligned}$$

where D^t_{Ins}, D^t_{For}, D^t_{Ind}, and R^t are also standardized to its series.

The outcomes of the VAR test are shown in Tables 6 and 7. The coefficients of cross-sectional averages and adjusted R^2 values are listed. The results for price consensus and return are presented in Panel A, and observations for net transaction amount and return are shown in Panel B. When comparing the two VAR results, the adjusted R^2 values of the first model are larger than the second model for all four regressions in both tables. This implies that our market consensus measure better explains its series and return series than when only utilizing data on net transaction amounts.

In Panel A of Table 6, we can see that price consensuses of all investors are positively explained by their one lagged series with coefficients above 0.3. Furthermore, past price consensus series of each agent positively affect the other's price consensus. This implies short-term information spillover, and we can check that spillover mostly occurs from

Table 5
Coefficients of correlations, autocorrelations, and cross-correlations in 2008.

Panel A. Correlation tests on investors' price consensuses and stock returns

	Institution's price consensus					Foreigner's price consensus					
Lag (period)	5	4	3	2	0	5	4	3	2	1	0
Inst. price consensus	0.22***	0.27***	0.33***	0.42***	1.00***	0.20***	0.22***	0.25***	0.32***	0.37***	0.35***
For. price consensus	0.24***	0.28***	0.31***	0.36***	0.35***	0.26***	0.30***	0.36***	0.44***	0.55***	1.00***
Ind. price consensus	0.36***	0.40***	0.43***	0.46***	0.15***	0.37***	0.41***	0.45***	0.47***	0.45***	0.12***
Stock return	−0.13***	−0.14***	−0.17***	−0.15***	0.33***	−0.12***	−0.13***	−0.17***	−0.18***	−0.16***	0.28***

Panel B. Correlation tests on investors' net transaction amounts and stock returns

	Institution's net transaction						Foreigner's net transaction					
Lag (period)	5	4	3	2	1	0	5	4	3	2	1	0
Inst. net transaction	0.02**	0.04***	0.10***	0.15***	0.28***	1.00***	−0.04***	−0.06***	−0.09***	−0.09***	−0.13***	−0.42***
For. net transaction	−0.01	−0.03***	−0.04***	−0.08***	−0.12***	−0.42***	0.04***	0.05***	0.08***	0.12***	0.24***	1.00***
Ind. net transaction	−0.01	−0.02*	−0.04***	−0.06***	−0.14***	−0.45***	0.00	0.00	0.00	−0.04***	−0.11***	−0.54***
Stock return	−0.02**	−0.02**	−0.02**	0.01	0.04***	0.31***	−0.01	−0.01*	−0.03***	−0.04***	−0.01	0.24***

*** Significant at 0.1%.
** Significant at 1%.
* Significant at 5%.

Table 5 (*continued*)

Panel A. Correlation tests on investors' price consensuses and stock returns

	Individual's price consensus						Stock return					
Lag (period)	5	4	3	2	1	0	5	4	3	2	1	0
Inst. price consensus	0.12***	0.15***	0.16***	0.19***	0.22***	0.15***	0.07***	0.09***	0.09***	0.17***	0.19***	0.33***
For. price consensus	0.12***	0.16***	0.19***	0.23***	0.25***	0.12***	0.09***	0.10***	0.12***	0.14***	0.17***	0.28***
Ind. price consensus	0.26***	0.30***	0.36***	0.42***	0.54***	1.00***	0.09***	0.10***	0.09***	0.05***	-0.01	-0.33***
Stock return	−0.10***	0.10***	−0.13***	−0.14***	−0.20***	−0.33***	−0.05***	−0.04***	−0.08***	0.00	0.03***	1.00***

Panel B. Correlation tests on investors' net transaction amounts and stock returns

	Individual's net transaction						Stock return					
Lag (period)	5	4	3	2	1	0	5	4	3	2	1	0
Inst. net transaction	0.03***	0.02**	0.00	−0.04***	−0.11***	−0.45***	−0.02*	−0.01	−0.01	0.07***	0.10***	0.31***
For. net transaction	−0.03***	−0.03***	−0.04***	−0.05***	−0.12***	−0.54***	0.01	0.01	0.02***	0.03***	0.06***	0.24***
Ind. net transaction	0.00	0.01*	0.05***	0.10***	0.26***	1.00***	0.00	0.00	−0.02***	−0.10***	−0.18***	−0.54***
Stock return	0.03***	0.04***	0.05***	0.03***	−0.03***	−0.54***	−0.05***	−0.04***	−0.08***	0.00	0.03***	1.00***

Table 6
Vector auto-regressive results in 2014.

Panel A. VAR results of investors' price consensuses and stock returns

		Institutional investor's consensus					Foreign investor's consensus					Individual investor's consensus					Return					Adj. R^2
	α	β_1^{Ins}	β_2^{Ins}	β_3^{Ins}	β_4^{Ins}	β_5^{Ins}	β_1^{For}	β_2^{For}	β_3^{For}	β_4^{For}	β_5^{For}	β_1^{Ind}	β_2^{Ind}	β_3^{Ind}	β_4^{Ind}	β_5^{Ind}	λ_1	λ_2	λ_3	λ_4	λ_5	
μ^t_{Ins}	−0.01	0.35***	0.11***	0.02	0.06***	0.03*	0.05***	0.03*	−0.02	0.02	−0.01	0.07***	0.04**	−0.02	0.03*	0.01	0.14***	0.04***	0.03**	0.01	0.01	0.363
μ^t_{For}	−0.01	0.07***	0.06***	0.02	0.02	0.01	0.33***	0.11***	0.06***	0.05***	0.01	0.06***	0.05***	0.01	0.01	-0.01	0.01	0.07***	0.03*	0.01	0.02*	0.385
μ^t_{Ind}	−0.01	0.24***	0.06***	0.03*	−0.02	0	0.21***	0.05***	0.03**	−0.02	0.01	0.36***	0.08***	0.06***	0.01	0.02*	0.08***	0	−0.01	0.01	−0.01*	0.526
R^t	0	−0.07***	0.02	−0.03	0.01	0.03	−0.11***	0	−0.02	0.01	0	−0.17***	0.02	−0.02	0.01	0.02	−0.07***	−0.02	0.01	0.01	0.02	0.048

Panel B. VAR results of investors' net transaction amounts and stock returns

		Institutional investor's net buying					Foreign investor's net buying					Individual investor's net buying					Return					Adj. R^2
	α	β_1^{Ins}	β_2^{Ins}	β_3^{Ins}	β_4^{Ins}	β_5^{Ins}	β_1^{For}	β_2^{For}	β_3^{For}	β_4^{For}	β_5^{For}	β_1^{Ind}	β_2^{Ind}	β_3^{Ind}	β_4^{Ind}	β_5^{Ind}	λ_1	λ_2	λ_3	λ_4	λ_5	
D^t_{Ins}	0	0.28***	0.08**	0	0.04*	0.02	−0.01	0	−0.04**	0	−0.01	0.06**	0.03	−0.03	0.02	0.02	0.09 ***	0	−0.01	−0.01	0.01	0.118
D^t_{For}	0	−0.08***	0.02	−0.01	0.01	−0.01	0.22***	0.08***	0.03*	0.05**	0	−0.03	0.03	−0.01	0.01	−0.03	−0.13***	0.04***	0.01	0.01	0.01	0.105
D^t_{Ind}	0	−0.03	−0.03	0.03	−0.03	0.01	−0.01	−0.02	0.04*	−0.02	0.03*	0.17***	0.01	0.07***	−0.02	0.03	0.02*	−0.02*	0	0.01	−0.02*	0.041
R^t	0	0.06***	0.02	−0.04	0	−0.01	0.01	0.01	−0.04*	0	−0.02	−0.02	0.05*	−0.01	0.03	0.03	−0.11***	−0.04***	−0.03**	−0.01	0.02	0.013

*** Significant at 0.1%.
** Significant at 1%.
* Significant at 5%.

Table 7
Vector auto-regressive results in 2008.

Panel A. VAR results of investors' price consensuses and stock returns

		Institutional investor's consensus					Foreign investor's consensus					Individual investor's consensus					Return					Adj. R^2
	α	β_1^{Ins}	β_2^{Ins}	β_3^{Ins}	β_4^{Ins}	β_5^{Ins}	β_1^{For}	β_2^{For}	β_3^{For}	β_4^{For}	β_5^{For}	β_1^{Ind}	β_2^{Ind}	β_3^{Ind}	β_4^{Ind}	β_5^{Ind}	λ_1	λ_2	λ_3	λ_4	λ_5	
μ^t_{Ins}	−0.02***	0.33***	0.10***	0.05**	0.01	0.01	0.09***	0.03*	−0.02	0.01	0.01	0.07***	0.05***	−0.01	0.01	0.00	0.12***	0.07***	−0.02	0.02	0.00	0.336
μ^t_{For}	−0.02***	0.10***	0.04**	0.00	0.01	0.03*	0.31***	0.11***	0.03	0.03*	0.04**	0.07***	0.06***	−0.01	0.00	0.00	0.13***	0.07***	0.03**	0.02*	−0.01	0.368
μ^t_{Ind}	−0.01	0.17***	0.06***	0.04**	0.02	−0.02	0.21***	0.05***	0.07***	0.02	−0.01	0.34***	0.05***	0.07***	0.02	−0.01	0.04***	0.01	−0.01	−0.02	−0.02**	0.473
R^t	0.00	−0.06***	0.02	0.00	0.01	−0.01	−0.11***	−0.01	−0.02	0.02	0.00	−0.14***	0.02	−0.06***	0.00	−0.04**	0.02	0.01	−0.07***	−0.02	−0.02**	0.069

Panel B. VAR results of investors' net transaction amounts and stock returns

		Institutional investor's net buying					Foreign investor's net buying					Individual investor's net buying					Return					Adj. R^2
	α	β_1^{Ins}	β_2^{Ins}	β_3^{Ins}	β_4^{Ins}	β_5^{Ins}	β_1^{For}	β_2^{For}	β_3^{For}	β_4^{For}	β_5^{For}	β_1^{Ind}	β_2^{Ind}	β_3^{Ind}	β_4^{Ind}	β_5^{Ind}	λ_1	λ_2	λ_3	λ_4	λ_5	
D^t_{Ins}	0.00	0.25***	0.07***	0.03	0.01	0.02	0.00	0.00	−0.04	0.00	0.01	0.03	0.03	−0.02	0.02	0.03	0.02*	0.03***	−0.04***	0.00	0.00	0.087
D^t_{For}	0.00	−0.06***	−0.02	−0.02	−0.01	0.02	0.16***	0.05**	0.02	0.00	0.02	−0.04*	0.01	−0.02	−0.02	0.00	0.03***	0.02	0.01	−0.01	−0.01	0.065
D^t_{Ind}	0.00	0.03	0.00	0.02	0.01	−0.03	0.06**	0.00	0.05*	0.01	−0.03	0.25***	0.01	0.07***	0.01	−0.03	−0.06***	−0.05***	0.02*	0.01	0.00	0.071
R^t	0.01	0.00	0.01	0.00	−0.01	0.00	−0.04*	−0.01	−0.01	0.00	0.00	−0.08***	0.02	−0.03	0.00	0.01	−0.01	0.00	−0.10***	−0.03***	−0.05***	0.015

*** Significant at 0.1%.
** Significant at 1%.
* Significant at 5%.

institutional investors' consensus and individual investors are mostly affected by the spillover. In addition, price consensus series are positively explained by the past returns, which means the market players set higher price consensus to the stock whose price is rising. The stock return is weakly negatively explained by the past price consensuses, which means the change of investor's price consensus gives weak negative effects. The vector autoregressive results in 2008 in Panel B of Table 7 show generally similar results as the normal period of 2014 except for few coefficients. Especially, foreign investors' price consensus is largely affected by the one lagged stock return in crisis period. This implies that foreign investors reacted more sensitively to the short term stock return in the crisis period.

The herding tendency, which is the one-lagged explanation of the same times series as in Panel A of Tables 6 and 7, also exists in the net transaction amount in Panel B of Tables 6 and 7. However, most of the other coefficients are insignificant and poorly explains the model.

3.5. Interpretation of the results based on previous studies

While the analyses in this paper are based on the proposed measure for price consensus for institutional, foreign, and individual investors, our findings support many of the previous studies on investor behavior.

As we have discussed throughout Section 3, the implications from the price consensus measure could be divided to long-term (e.g., quarterly) and short-term (e.g., daily) behaviors. We interpret the long-term implication as investors' fundamental view on a risky asset such as stocks and short-term implication as daily fluctuations on investors' view due to short-term events such as cash-flow news. Our results reveal that, in the long term, institutional and foreign investors usually have distinct views on stocks, and individual investor's view lies in between the two (i.e., individuals act as uninformed neutral traders).

In the short term, we find that all investors' price consensuses are highly auto-correlated, which might imply positive feedback trading and herding behavior of the investors as argued by Wermers (1999). In addition, institutional and foreign investors' daily price consensuses on the same day are highly correlated, but individual investors have contrarian consensus as shown by Grinblatt and Keloharju (2000).

The high correlation between individual investors' price consensus and the past institution and foreign investors' price consensus might imply information spillover from institution and foreign investors to individual investors, consistent with the individuals' under reaction behavior to cash-flow news as explained in Cohen et al. (2002). Also, the information spillover might cause the higher future market impact of institutional investors than individual's as in Nofsinger and Sias (1999). Recently, Han and Yang (2013) model the information spillover from the rational informed investors to rational less informed investors by social communication and found out the incentives to “free ride” would exists. The results of this study is consistent to our result that short-term information spillover occurs from institutions and foreign investors to individual investors, even though our model assumes information or price consensus as independent. This spillover phenomenon might amplify the price impact of the transactions of institutional and foreign investors.

In addition, similar to momentum behavior of mutual funds in Grinblatt et al. (1995), foreign investors in Grinblatt and Keloharju (2000), and institutional investors in Griffin et al. (2003), we found that all investors, institutional, foreign and individual investors, are momentum investors to some degree, whose daily price consensus rises to past winners with high returns. However, the future return is negatively correlated to the price consensus changes.

4. Conclusion

Examining the behavior of investors is critical for understanding the dynamics of the financial market. Thus, there has been various studies that model investment behavior. This paper concentrates on the dynamics among three major types of investors: institutional, foreign, and individual investors. In order to provide a more thorough analysis, a measure on price consensus is presented along with its theoretical derivation. The main contribution of this paper is proposing the price consensus measure and demonstrating its advantage empirically when compared against previous approaches that only exploit information from net transaction amounts.

The effectiveness of the price consensus measure is tested using historical data of the Korean stock market due to the availability of data on stocks and investor transactions. Several analyses are performed, including correlation tests and vector auto-regression tests, for the constituents stocks of the KOSPI 100 index in 2008 and 2014. The results reveal that institutional and foreign investors tend to have long-term views that are either the highest or the lowest while individual investors tend to have views in between the two. This indicates that institutions and foreigners are more self-initiated traders in the long-term. In addition, foreign investors usually underestimated the price during the financial crisis period of 2008. The findings on short-term views illustrate that all three investor types are momentum investors and evidence of spillover effects are also observed. Finally, the observations obtained from the price consensus measure exhibits many implications that match prior studies, which confirms the strength of the proposed measure as well as its capability of modeling various aspects of investment behavior.

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Appendix A

Proof of Proposition 1. (a) Optimal θ_i that maximize the likelihood function given $\hat{\gamma}_i$

$$Likelihood = \prod_i \prod_t N\left(D_i \middle| \frac{\theta_i - p^t}{\hat{\gamma}_i \sigma^2}, \frac{s_i^2}{\hat{\gamma}_i^2 \sigma^4}\right)$$

$$\theta_i = \underset{\theta_i}{\operatorname{argmax}} \prod_i \prod_t N\left(D_i \middle| \frac{\theta_i - p^t}{\hat{\gamma}_i \sigma^2}, \frac{s_i^2}{\hat{\gamma}_i^2 \sigma^4}\right)$$

$$= \underset{\theta_i}{\operatorname{argmax}} \sum_t \ln N\left(D_i^t \middle| \frac{\theta_i - p^t}{\hat{\gamma}_i \sigma^2}, \frac{s_i^2}{\hat{\gamma}_i^2 \sigma^4}\right)$$

$$= \underset{\theta_i}{\operatorname{argmax}} \sum_t -\frac{1}{2} \frac{\left(D_i^t - \frac{\theta_i - p^t}{\hat{\gamma}_i \sigma^2}\right)^2}{\frac{s_i^2}{\hat{\gamma}_i^2 \sigma^4}}$$

$$= \underset{\theta_i}{\operatorname{argmax}} \sum_t -\frac{\left(D_i^t \hat{\gamma}_i \sigma^2 - (\theta_i - p^t)\right)^2}{s_i^2}$$

Since the above equation is a convex function, the point where the first order condition is satisfied is the optimum.

$$\text{First order condition}: \sum_t \frac{2\left(D_i^t \hat{\gamma}_i \sigma^2 - (\theta_i - p^t)\right)}{s_i^2} = 0$$

$$\theta_i = \frac{\sum_t p^t}{T} + \frac{\sum_t D_i^t}{T} \hat{\gamma}_i \sigma^2$$

(b) Optimal $\hat{\gamma}_i$ that maximize the likelihood function is

$$\textit{Likelihood function} = \prod_i \prod_t N\left(D_i | \frac{\theta_i - p^t}{\hat{\gamma}_i \sigma^2}, \frac{s_i^2}{\hat{\gamma}_i{}^2 \sigma^4}\right)$$

$$\hat{\gamma}_i = \underset{\hat{\gamma}_i}{\operatorname{argmax}} \prod_i \prod_t N\left(D_i | \frac{\theta_i - p^t}{\hat{\gamma}_i \sigma^2}, \frac{s_i^2}{\hat{\gamma}_i{}^2 \sigma^4}\right)$$

$$= \underset{\hat{\gamma}_i}{\operatorname{argmax}} \sum_t \ln N\left(D_i^t | \frac{\theta_i - p^t}{\hat{\gamma}_i \sigma^2}, \frac{s_i^2}{\hat{\gamma}_i{}^2 \sigma^4}\right)$$

$$= \underset{\hat{\gamma}_i}{\operatorname{argmax}} \sum_t \ln \hat{\gamma}_i - \frac{\sum_t \left(D_i^t - \frac{\theta_i - p^t}{\hat{\gamma}_i \sigma^2}\right)^2}{2 \frac{s_i^2}{\hat{\gamma}_i{}^2 \sigma^4}}$$

$$= \underset{\hat{\gamma}_i}{\operatorname{argmax}}\, T \ln \hat{\gamma}_i - \sum_t \frac{\left(D_i^t - \frac{\frac{\sum_t p^t}{T} + \frac{\sum_t D_i^t}{T} \hat{\gamma}_i \sigma^2 - p^t}{\hat{\gamma}_i \sigma^2}\right)^2}{2 \frac{s_i^2}{\hat{\gamma}_i{}^2 \sigma^4}}$$

$$= \underset{\hat{\gamma}_i}{\operatorname{argmax}}\, T \ln \hat{\gamma}_i - \sum_t \frac{\left(\hat{\gamma}_i \sigma^2 D_i^t - \overline{p} - \overline{D}_i \hat{\gamma}_i \sigma^2 + p^t\right)^2}{2 s_i^2}$$

$$= \underset{\hat{\gamma}_i}{\operatorname{argmax}}\, T \ln \hat{\gamma}_i - \sum_t \frac{\left(\hat{\gamma}_i \sigma^2 (D_i^t - \overline{D}_i) + (p^t - \overline{p})\right)^2}{2 s_i^2}$$

where $\overline{D}_i$ and $\overline{p}$ are the mean values of D_i^t and p^t during the period, and T is total time period.

$$\text{Let} \quad f(\hat{\gamma}_i) := \ln \hat{\gamma}_i - \sum_t \frac{\left(\hat{\gamma}_i \sigma^2 (D_i^t - \overline{D}_i) + (p^t - \overline{p})\right)^2}{2 s_i^2}$$

$$\frac{d^2 f(\hat{\gamma}_i)}{d \hat{\gamma}_i{}^2} = -\frac{T}{\hat{\gamma}_i^2} - \sum_t \frac{\left(\sigma^2 (D_i^t - \overline{D}_i)\right)^2}{s_i^2}.$$

When $\hat{\gamma}_i > 0$, the second derivative is negative, so the function is a convex function on the convex domain $\hat{\gamma}_i > 0$. Therefore, we can conclude that the optimal point is the point that satisfies the first order condition,

$$\frac{df(\hat{\gamma}_i)}{d\hat{\gamma}_i} = \frac{\mathrm{T}}{\hat{\gamma}_i} - \sum_t \frac{\left(\hat{\gamma}_i \sigma^2 (D_i^t - \overline{D}_i) + (p^t - \overline{p})\right) \sigma^2 (D_i^t - \overline{D}_i)}{s_i^2} = 0.$$

The solution of the above equation can be expressed as

$$\hat{\gamma}_i = \left[\begin{array}{l} \dfrac{-\sum_t (D_i^t - \overline{D}_i)(p^t - \overline{p}^t) - \sqrt{\left(\sum_t (D_i^t - \overline{D}_i)(p^t - \overline{p})\right)^2 + 4T s_i^2 \sum_t (D_i^t - \overline{D}_i)^2}}{2\sigma^2 \sum_t \left(D_i^t - \overline{D}_i^t\right)^2} \\ \dfrac{-\sum_t (D_i^t - \overline{D}_i)(p^t - \overline{p}^t) + \sqrt{\left(\sum_t (D_i^t - \overline{D}_i)(p^t - \overline{p})\right)^2 + 4T s_i^2 \sum_t (D_i^t - \overline{D}_i)^2}}{2\sigma^2 \sum_t (D_i^t - \overline{D}_i)^2} \end{array} \right.$$

However, we can easily notice that the first equation is negative regardless of whether the value of $\sum_t (D_i^t - \overline{D}_i)(p^t - \overline{p})$ is negative or positive. Therefore, the solution on the domain $\hat{\gamma}_i > 0$ becomes

$$\hat{\gamma}_i = \frac{-\sum_t (D_i^t - \overline{D}_i)(p^t - \overline{p}) + \sqrt{\left(\sum_t (D_i^t - \overline{D}_i)(p^t - \overline{p})\right)^2 + 4T s_i^2 \sum_t (D_i^t - \overline{D}_i)^2}}{2\sigma^2 \sum_t (D_i^t - \overline{D}_i)^2}.$$

(c) By Eq. (1)

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