Multi-Stage Financial Planning Models: Integrating Stochastic Programs and Policy Simulators

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Abstract

This chapter reviews multi-stage financial planning models, with a focus on practical approaches for optimizing investors' performance over time. We discuss two major frameworks for constructing financial planning models: 1) policy rule simulation and optimization; and 2) multi-stage stochastic programming. We advocate an integrated approach, in which a stylized stochastic program helps the investor discover robust decision/policy rules. In the second stage, the policy optimizer compares policy rules as well as provides detailed information about future investment performance. To illustrate benefits, we apply the dual strategy to defined benefit pension plans in the United States.

I. Introduction

Multi-stage financial planning models offer significant advantages over the classical single-stage (static) portfolio approaches. First, importantly, investment performance is enhanced by capturing "rebalancing gains" when a portfolio is modified. Rebalancing a portfolio can be considered as an option – to be exercised when adding value to the investors' performance. Second, a multi-stage model can address many real-world issues in considerable detail -- such as transaction-market impact costs, dynamic correlation structures, integrating assets with liability and goals over time, and decisions that are conditioned on the state of the economy. These issues are difficult to model within a single period framework. In a similar fashion, a multi-stage model can readily depict temporal goals and objectives. For instance, we can include stopping rules that activate when a target wealth value is reached. See Dantzig and Infanger (1993), Consigli and Dempster (1998), Mulvey and Ziemba (1998), and Mulvey et al. (2003) for examples. A theoretical discussion can also be found in Fernholz and Shay (1982) and Luenberger (1998).

Unfortunately, multi-stage financial models possess two significant interrelated drawbacks: 1) they give rise to complex models and require extensive computational resources, and 2) the modeling recommendations are difficult to understand by decision makers. While there are numerous stochastic programming applications appearing in the

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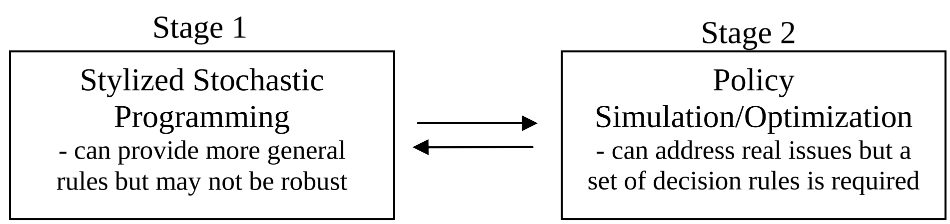
academic field, many investors are simply unable to grasp the complexity of a multi-stage stochastic program, and accordingly, are often reluctant to "trust" the model's recommendations for assigning their capital.

To overcome these barriers, we advocate a dual strategy illustrated in Figure 1. In the first stage of the dual approach, investors apply a simplified (stylized) stochastic program to generate an optimal set of decisions defined over a scenario tree. Since decisions can occur anywhere in the feasible region as defined by the constraints, the stochastic program will offer general recommendations. However, by its nature, a stochastic program must present a simplified view of the real-world, which may lead to a non-robust optimal solution.

Therefore, in the second stage, a more detailed policy simulation-optimization approach is adopted. Unlike the stochastic program with generic decisions, the decisions in a policy simulator must be a function of a formal policy/decision rule. Also, the policy rules cannot employ information that is unavailable to the investor at a given time period; the model must enforce non-anticipativity. Since these approaches are complementary to each other, the dual strategy can not only help create a set of new policy rules but also evaluate the rules with respect to robustness.

Figure 1
Illustration of the Dual Strategy



To illustrate, we apply the dual strategy to an important problem in the area of asset and liability management -- U.S. defined benefit (DB) pension trusts. The major goals of the integrated ALM model for DB pension trusts are to satisfy the interests of several entities – shareholders, sponsoring company, retirees and regulators (public) – which may conflict each other. For instance, for the shareholders, the goal should be to maximize value for the company, minimize present value (and volatility) of contribution for the pension trust. Providing an adequate level of safety for the pension plan and future beneficiaries would be the main objectives for the retirees. On the other hand, the regulators would minimize the possibility of the bankruptcy and plan termination. Therefore, there goals are best satisfied by constructing a multi-objective model whose output consists of a set of efficient frontiers.

There are numerous examples of stochastic programs implemented in different countries. A noteworthy early application is the Russell system for the Yasuda Insurance Company in Japan (Cariño et al., 1994). Mulvey et al. (2000) depicts the details of Towers Perrin

CAP:Link system. Geyer et al. (2005) describe the financial planning model InnoALM developed by Innovest for Austrian pension funds. Hilli et al. (2003) develop a stochastic program for a Finnish pension company; their empirical study indicates that the results under the stochastic program outperform those under traditional fixed-mix strategies. Pflug et al. (2000) developed a modular decision support tool for asset-liability management – the AURORA Financial Management System at the University of Vienna. Pflug and Swietanowski (2000) also show that the non-convexity of the ALM model can be resolved by approximating it to a convex problem. Dutch researchers have also achieved success in implementing ALM models for pension planning. Boender et al. (1998) describe the ORTEC model. The doctoral dissertation of Dert (1995) presents a scenario-based model for analyzing investment/funding policies of Dutch defined benefit pension plans. Ziemba and Mulvey (1998), Arbeleche et al. (2003), Mulvey et al. (2004a), Kouwenberg and Zenios (2006), Dempster et al. (2006) and Zenios and Ziemba (2006, 2007) provide examples of related implementations. Also, Ziemba and Mulvey (1998) and Ziemba (2003) include examples of successful asset and liability management (ALM) systems within a multi-period setting. Fabozzi et al. (2004) discuss applications of ALM in European pension funds.

Several researchers have studied ALM problems in a continuous-time setting. Chapter 5 of Campbell and Viceira (2002) provide an introduction to strategic asset allocation in continuous time. A group of ALM models extend the early approach of Merton (1973). Rudolf and Ziemba (2004) present such a continuous time model for pension fund management. Although stochastic-control models have difficulty in incorporating practical and legal constraints of pension plans, they possess the advantage of producing intuitive closed-form solutions or solving the system of equations satisfying optimality conditions numerically in many cases.

The remainder of the chapter is organized as follows. The next section illustrates the advantage of multi-period models for the asset-only investors to improve asset allocation decision with several practical examples. In the section III, we apply the dual strategy to the DB pension problems in the United States. The final section provides conclusions and future research topics.

II. Multi-Period Models for Asset-only Allocation

This section illustrates the advantages of applying multi-period portfolio approaches. We begin with the well-know fixed-mix investment rule due to its simplicity and profitability. Also, we provide several prominent examples that employ the fixed-mix approach for asset-only investors. This policy rule serves as a benchmark both for other types of rules and for the recommendations of a stochastic programming model.

II.1 Fixed Mix Policy Rules

First, we describe the performance advantages of the fixed-mix rule over a static, buy-and-hold perspective. This rule generates greater return than the static model by means of rebalancing. The topic of re-balancing gains (also called excess growth or volatility pumping) as derived from the fixed-mix decision rule is well understood for a theoretical perspective. The fundamental solutions were developed by Merton (1969) and Samuelson (1969) for long-term investors. Further work was done by Fernholz and Shay (1982), and Fernholz (2002). Luenberger (1998) presents a clear discussion. We illustrate how rebalancing the portfolio to a fixed-mix creates excess growth. Suppose that a stock price process P_t is lognormal so it can be represented by the equation

$$dP_t = \alpha P_t dt + \sigma P_t dz_t \tag{1}$$

where α is the rate of return of P_t and σ^2 is its variance, z_t is Brownian motion with mean 0 and variance t.

The risk-free asset follows the same price process with rate of return equal to r and standard deviation equal to 0. We represent the price process of risk-free asset by B_t:

$$dB_t = rB_t dt \tag{2}$$

When we integrate the equation (1), the resulting stock price process is

$$P_t = P_0 \, \mathrm{e}^{(\alpha - \sigma^2/2)t + \sigma z_t} \tag{3}$$

It is well documented that the growth rate $\gamma = \alpha - \sigma^2/2$ is the most relevant measure for long-run performance. For simplicity, we assume equality of growth rates across all assets. This assumption is not required for generating excess growth, but it makes the illustration easier to understand.

Next, let's assume that the market consists of n stocks, each with stock price processes $P_{1,t}, ..., P_{n,t}$ following the lognormal process. A fixed-mix portfolio has a wealth process W_t that can be represented by the equation

$$dW_t / W_t = \eta_1 \, dP_{1,t} / P_{1,t} + ... + \eta_n \, dP_{n,t} / P_{n,t} \tag{4}$$

where $\eta_1, ..., \eta_n$ are the fixed weights given to each stock (proportion of capital allocated to each stock). In this case, the weights sum up to one

$$\sum_{i=1}^{n} \eta_i = 1 \tag{5}$$

The fixed-mix strategy in continuous time always applies the same weights to stocks over time. The instantaneous rate of return of the fixed-mix portfolio at anytime is the weighted average of the instantaneous rates of returns of the stocks in the portfolio.

In contrast, a buy-and-hold portfolio is one where there is no rebalancing and therefore the number of shares for each stock remains constant over time. This portfolio can be represented by the wealth process W_t :

$$dW_t = m_1 dP_{1,t} + ... + m_n dP_{n,t} \tag{6}$$

where $m_1, ..., m_n$ depicts the number of shares for each stock.

Again for simplicity, let's assume that there is one stock and a risk-free instrument in the market. This case is sufficient to demonstrate the concept of excess growth in a fixed-mix portfolio as originally presented in Fernholz and Shay (1982). Assume that we invest η portion of our wealth in the stock and the rest (1-η) in the risk-free asset. Then the wealth process W_t with these constant weights over time can be expressed as

$$dW_t / W_t = \eta\, dP_t / P_t + (1-\eta)\, dB_t / B_t \tag{7}$$

where P_t is the stock price process and B_t is the risk-free asset.

When we substitute the dynamic equations for P_t and B_t, we get

$$dW_t / W_t = (r + \eta(\alpha - r))dt + \eta\sigma dz_t \tag{8}$$

Assuming the growth rate of all assets in the ideal market should be the same over long-time periods, the growth rate of the stock and the risk-free asset should be equal. Hence

$$\alpha - \sigma^2 / 2 = r \tag{9}$$

From equation (8), we can see that the rate of return of the portfolio, α_ω, is

$$\alpha_w = r + \eta(\alpha - r) \tag{10}$$

By using (9), this rate of return is equal to

$$\alpha_w = r + \eta\, \sigma^2 / 2 \tag{11}$$

The variance of the resulting portfolio is

$$\sigma_w^2 = \eta^2 \sigma^2 \tag{12}$$

Hence the growth rate of the fixed-mix portfolio becomes

$$\gamma_w = \alpha_w - \sigma_w^2 / 2 = r + (\eta - \eta^2)\sigma^2 / 2 \tag{13}$$

This quantity is greater than r for $0 < \eta < 1$. As it is greater than r, which is the growth rate of individual assets, the portfolio growth rate has an excess component, which is $(\eta - \eta^2)\sigma^2/2$. Excess growth is due to rebalancing the portfolio constantly to the target fixed-mix. The strategy moves capital out of stock when it performs well and moves capital into stock when it performs poorly. By moving capital between the two assets in the portfolio, a higher growth rate than each individual asset is achievable. It can be shown that the buy-and-hold investor with equal returning assets lacks the excess growth component. Therefore, buy-and-hold portfolios under-perform fixed-mix portfolios in various cases. We can easily see that the excess growth component is larger when σ takes a higher value.

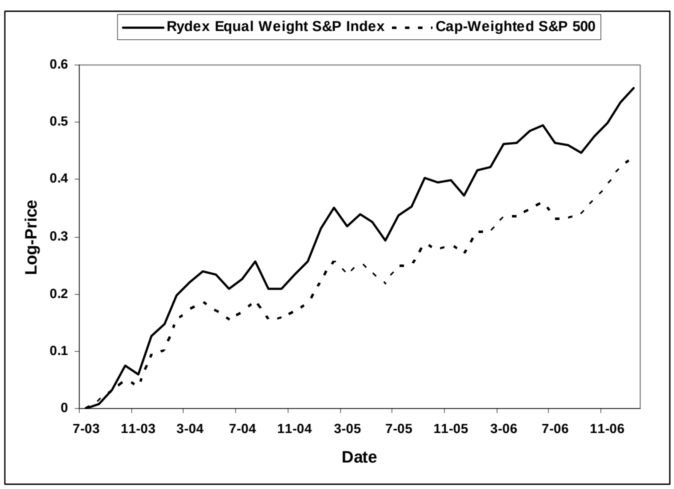
II.2 Examples of Fix-Mix Policy Rules

Many investors have applied versions of the fixed-mix rules with practical successes. For example, the famous 60/40 norm (60% equity and 40% bonds) falls under this policy. Here, at each period, we rebalance the portfolio to 60% equity and 40% bond. Another good example is S&P 500 equal-weighted index (S&P EWI) by Rydex Investments (Mulvey, 2005). As opposed to traditional cap-weighted S&P index, stocks have the same weight (1/500) and the index is rebalanced semi-annually to maintain the weights over time. To illustrate the benefits of applying the fixed mix policy rule, during 1994~2005, S&P EWI achieved 2% excess return with only 0.6% extra volatility compared to S&P 500 index. Figure 2 illustrates log-prices of S&P 500 and S&P EWI for last 4 years[3].

Figure 2

Log-Prices of S&P 500 Index and S&P EWI during Jul.2003~Dec.2006

This figure illustrates the log price processes for S&P EWI and S&P 500 from Jul.2003 to Dec.2006. Each index is scaled to have a log-price of 0 at the beginning of the sample period. In term of the total return, S&P EWI outperformed S&P 500 index for last 4 years and this performance difference between 2 assets can be interpreted as a rebalancing gain due to the fixed mix policy rule.

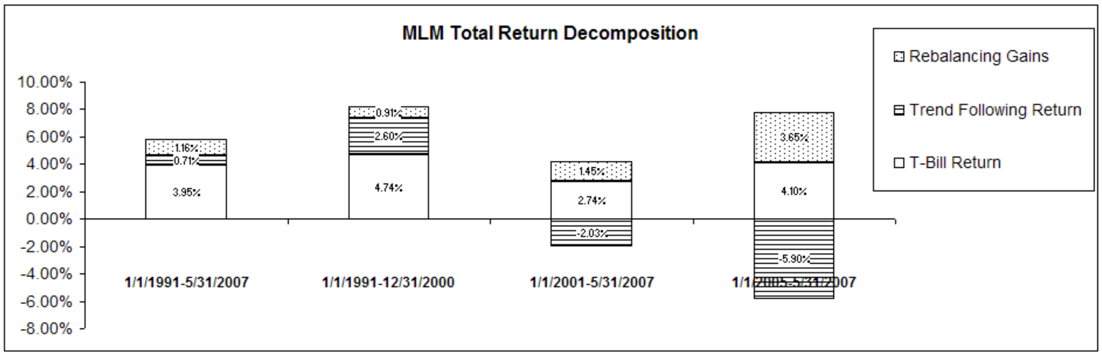


[3] The advantages of the equal weighted S&P 500 index is partially due to rebalancing gains and partially due to the higher performance of mid-size companies over the discussed period.

Figure 3

Decomposition of MLM Index returns for different time periods

The total return of the MLM index can be decomposed into three parts – 1) trend following returns, 2) T-bill returns and 3) rebalancing gains. Rebalancing portfolio provided positive returns over 1991-2007.



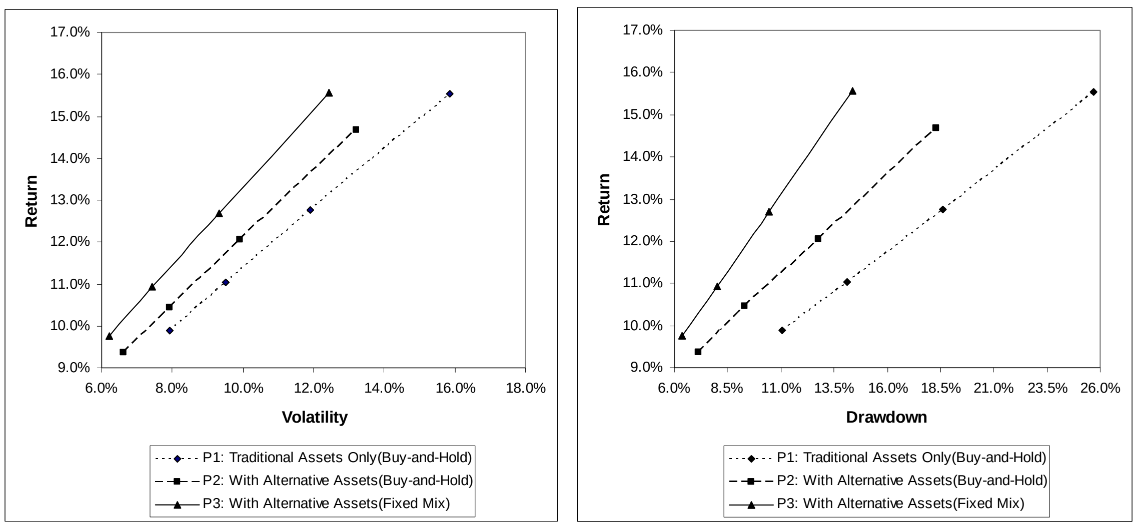
Another significant example involves the Mount Lucas Management (MLM) Index (Mulvey et al., 2004b). It is an equally weighted, monthly rebalanced investment in twenty-five futures contracts in commodity, fixed income, and currency markets. Briefly, the monthly positions (long or short) are determined by trend following strategies (long or short depending upon a trend line). The total return of the MLM Index can be decomposed into three parts. The first one is the T-bill return gained from the capital allocated for margin requirements. The second component is generated by trend following the futures prices. The third component, rebalancing gains, is earned when all markets are invested with fixed weights at the beginning of each month. If trend following strategy had been applied to all the markets without reweighing at each month, then there would be no rebalancing gains. Figure 3 shows how those three components affected the total return of the MLM Index for selected time periods. Trend following has underperformed its long term averages for the last several years. Still, rebalancing provided positive returns over the recent period. These empirical results show how periodic reallocation of capital among diverse markets boosts the performance of a long term investment strategy with the contribution of rebalancing gains.

Mulvey and Kim (2007a) illustrate how the fixed mix rule can improve performance in the realm of alternative investments. Their analysis constructs three portfolios: (P1) a buy-and-hold portfolio of only traditional assets, (P2) a buy-and-hold portfolio of traditional and alternative assets, (P3) a fixed mix portfolio of both traditional and alternative assets. In this regard, they employ the fixed-mix rule at two levels. First at the stock selection level, they substitute an equal-weighted S&P 500 index for the capital-weighted S&P 500 fund. The equal weighted index has generated better performance over the standard S&P500 index, as would be expected due to the additional returns gotten from re-balancing the mix. Then, the portfolio is rebalanced monthly to fulfill the fixed mix policy rule at the asset selection level. Figure 4 illustrate the investment performance of each of three portfolios. Note that several degrees of leverage – at several values: 20%, 50%, and 100% are applied to each portfolio. From the figure, it is evident that applying fixed mix rules can improve investment performance.

Figure 4
Efficient Frontiers of the Portfolios with/without Alternative Assets

Left figure illustrates efficient frontiers in volatility-return plane, while right one is drawn in maximum drawdown-return plane. The efficient frontier of P3 contains those of P1 and P2 in both cases, which clearly exhibit the role of alternative assets in portfolio construction.

Portfolio	Description	Constituents
P1	Traditional Assets Only (Buy-and-Hold)	Traditional assets: SP500, LB Bond, EAFE, NAREIT, GSCI, STRIPS
P2	With Alternative Assets (Buy-and-Hold)	Traditional assets: SP500, LB Bond, EAFE, NAREIT, GSCI, STRIPS Alternative assets: Man Futures, Hedge Fund Ind., L/S Ind., Currency
P3	With Alternative Assets (Fixed Mix)	Traditional assets: *SP EWI*, LB Bond, EAFE, NAREIT, GSCI, STRIPS Alternative assets: Man Futures, Hedge Fund Ind., L/S Ind., Currency



The fixed-mix approach can also improve investment performance in the face of parameter uncertainty. A good example is the "dynamic diversification" strategy, which is a fixed mix portfolio consisting of various momentum strategies in several stock markets, suggested by Mulvey and Kim (2007b).

Herein, we develop a new decision rule based on momentum in equity markets. This rule is applied in conjunction with fixed-mix within a portfolio context. There is a considerable amount of literature on the predictability of stock returns based on its past performance. De Bondt and Thaler (1985, 1987) argue that the "loser" portfolio shows superior performance to the "winner" portfolio over the following 3 to 5 years. Lehmann (1990) and Jegadeesh (1990) illustrate short-term return reversals of "winner" and "loser" stocks. Also, Jegadeesh and Titman (1993) report the momentum of stock returns for the intermediate term (3 to 12 months). Rouwenhorst (1998), Kang et al. (2002), Demir et al. (2004) and Cleary et al. (1998) provide the evidence of the profitability of the momentum strategy in overseas stock markets. Among many variants of the momentum strategies such as the 52-week high momentum portfolio (George and Hwang, 2004) and the market index momentum portfolio (Chan et al, 2000), we examine the industry-level momentum strategy.

Beside the rebalancing gains, applying the fixed mix approach to various settings of the momentum strategy has another advantage. Although its profitability is widely accepted, the performance of strategies depends on parameter selection such as the evaluation period, the holding period, etc. However, by forming a portfolio of momentum strategies (via fixed-mix), the resulting portfolio is much less dependent on the parameters, thus provides robust performance.

To construct a "dynamic diversification" portfolio of equity positions, we first generate industry-level momentum strategies that follow the rules of the relative strength portfolio of Jegadeesh and Titman (1993). The implemented algorithm applies the following rules: At the beginning of the sample period, all industries defined as in Datastream service with return history of at least t_2-month are ranked based on their past t_1- and t_2-month returns ($t_1 < t_2$). Then industries in the first decile (four industries) from each t_1 and t_2 ranking lists are chosen to be put into the portfolio with the equal weight (eight industries total). If an industry is shown in the top decile for both past t_1 and t_2 months, we place a double-weight on it. The equal weighted portfolio is held for the given holding period (H) without any rebalancing and we repeat the industry-selecting process after H-month. We construct the momentum strategies with six different settings for (t_1, t_2) observation length pair – (3,6), (3,9), (3,12), (6,12), (6,18) and (12,18) – and four different holding periods – 3, 6, 9, 12. In our empirical study, we apply a scoring approach based on recent historical returns. These rules are applied in five non-overlapping regions – US, EU, Europe except EU, Japan and Asia except Japan – to potentially reduce the correlations over 1980 to 2007. Then, for each of four holding periods, we construct the dynamic diversification portfolio by applying the fixed mix policy rule to thirty strategies.

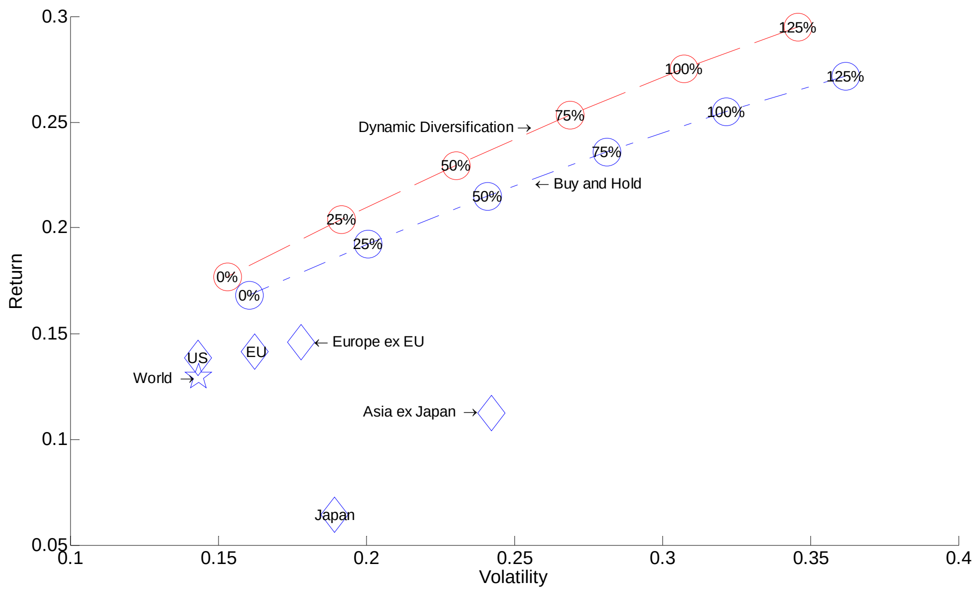
Here, we focus on industry level data for several reasons: 1) the globalization of trade and money flows leads many firms to broaden their exposure to international markets. These firms conduct their own diversification by a variety of approaches, such as moving production/manufacturing out of the home country, or international mergers. An example is the purchase of Chrysler by Mercedes Benz. The combined firm fits an industrial designation, but is difficult to categorize within a single country. (Germany or the U.S.?) 2) A second reason to focus on industries (rather than individual stocks) is to broaden the scope of momentum for institutional investors; there is greater capacity if the core assets depict industries as compared with individual stocks. 3) We believe that future asset allocation studies (as well as asset-liability management) will need to include industry exposure and concentration in order to provide improved risk analytics. 4) With introduction of alternative asset classes, such as ETF's, which enable us to trade industries easily, industries chosen by the momentum strategy provide for international diversification.

Figure 5 illustrates the investment performance of the dynamic diversification portfolio at several leverage levels. Not only the market indices but also the buy-and-hold portfolio is located on the right-bottom region compared to the dynamic diversification portfolio. In fact, among thirty industry-level momentum strategies that are utilized for the dynamic diversification portfolio, only one setting is located on the left-top part, which clearly indicates the robustness of its investment performance (See Figure A1 in the appendix).

Figure 5

Performance of Dynamic Diversification Portfolio with Leverage

Dynamic diversification portfolio is an equally weighted fixed mix portfolio of 30 momentum strategies -- five regions, six settings. Each number next to a point on the line represents leverage. 3-month U.S. T-bill is used. The sample period is 1980~2006.



II.3 Practical Issues Regarding Fixed-Mix Rules

From the derivation of the rebalancing gain, the desirable properties of assets in order to achieve rebalancing gain can be summarized as follows. First, suppose two assets in the derivation above are perfectly correlated. Then, it can be easily shown that the rebalancing gain is zero. From this, it is evident that diversification among assets plays a major role to achieve an excess growth rate. This observation suggests that dynamic diversification is essential in order to produce extra gains via multi-period approaches. Also, as always, diversification provides a source of reducing portfolio risk. Second, given a set of independent assets, the rebalancing gain ($(\eta-\eta^2)\sigma^2/2$) increases as the volatilities of assets increase. To benefit from rebalancing gain, the volatility of each asset should be reasonably high. In this context, the traditional Sharpe Ratio might not be a good measure for individual asset in terms of multi-period portfolio management, even though it remains valid at the portfolio level. Additionally, low transaction costs (fees, taxes, etc.) are desirable, since applying the fixed mix policy rule requires portfolio rebalancing. In summary, the properties of the best ingredients for the fixed mix rule are: 1) relatively good performance (positive expected return); 2) relatively low correlations among assets; 3) reasonably high volatility; and 4) low transaction costs.

Potential obstacles related to the fixed mix approach should be addressed. First, it is becoming harder to locate independent or low correlated assets. For instance, oil and corn, which were once relatively independent, are now highly correlated, because ethanol is manufactured from corn. Also, due to globalization, the correlation of assets across countries is becoming higher. Second, even if we are successful in finding a set of independent assets with positive returns and high volatilities, independence is likely to disappear under extreme conditions; there is much evidence that stock correlations dramatically increase (called contagion) when market crashes. Furthermore, it is well-known that stock returns and volatilities are negatively correlated. Third, since the fixed-mix model requires portfolio rebalancing, one must consider transaction costs, such as capital gain taxes. These circumstances suggest that other policy rules will outperform the fixed-mix rule. In the next section, we take up a systematic approach for discovering, and evaluating new policy rules.

III. Application of the Dual Strategy to ALM

In this section, we suggest that the dual strategy provides an ideal framework for assisting investors who must address future liabilities and goals. A prominent example involves the pension arena. Both the traditional defined benefit (DB) and the growing defined contribution (DC) plans have been successfully posed as multi-period models. The generic domain takes several names, including asset and liability management, dynamic financial analysis (DFA) for insurance companies, and enterprise risk management (ERM) in the corporate finance setting. As an additional advantage, a multi-period model helps investors understand the pros/cons of combining investment and savings (contribution) strategies.

A number of successful pension planning systems are based on elements of the dual strategy, including the Towers Perrin – Tillinghast system (Mulvey et al., 2000). Also, Mulvey and Simsek (2002) present a stochastic program for the asset allocation of a DB pension plan. This model takes the cash contributions by the sponsoring entity (company or non-profit agency) as given. Contributions arise from two major sources: plan sponsor's own capital and/or possible borrowing. Mulvey et al. (2008) extend the model by linking the pension plan with its sponsoring company. In addition to the asset allocation decisions, the integrated model addresses: 1) the sponsor's contribution policy and 2) the corporate borrowing policy for deciding whether to borrow funds and what proportion of the borrowing should be put into the pension plan. Zhang (2006) extends the integrated model by adding the company's dividend policy as the third corporate decision variable and enhances the corporate model by decomposing the company into a headquarters and several divisions. Earlier, Peskin (1997) argued that the economic cost of a defined-benefit pension plan is the present value of future contributions. In order to achieve savings, he showed that it is necessary for corporate sponsors 1) to integrate asset allocation and contribution policies and 2) to make significant changes in investment policies which include, among others, adopting an appropriate rebalancing rule.

We provide an example of the defined benefit pensions for the telecommunication and the automobile sectors to illustrate the idea of the dual strategy. Since the passage of the Employee Retirement Income Security Act (ERISA) in 1974, the DB pension system in the United States has grown from a modest level of assets (less than $100 Billion) to over $5 Trillion in 2004. Unfortunately, numerous DB pension trusts in the U. S. (and elsewhere) have seen their adequate surpluses disappear during the period 2000 to 2003. Simply put, a pension plan is judged to be healthy if the plan's funding ratio (market value of assets to discounted value of liabilities) is greater than 100%. A plan with a low funding ratio, less than 80-90%, is called under-funded, leading to several required steps by the sponsoring company, including typically a contribution to the pension trust. A fully funded trust has a ratio equal to 100%.

The telecommunication sector in the United States possesses a large pension system relative to the sector's market capitalization. Moreover, it has been and will likely continue to encounter slow growth. Fortunately, the current overall funding ratio is reasonable – roughly 90% in 2005. As an initial step, we set up an anticipatory multi-stage stochastic program. To simplify the analysis, we treat the sector as a single aggregate company. We employ a scenario generator that has been in use by one of the major actuarial firms for over a decade. The stochastic program consists of 1000 scenarios over a nine-year horizon (several hundred thousand nonlinear variables and constraints). We have found that a stochastic program consisting of 1000 scenarios is a reasonable compromise between model realism and computational costs for pension planning problems. In most cases, we advocate that the planning model be rerun on a recurring basis, at least once per year or more often if large changes take place in the markets. The scenario generator consists of a set of cascading stochastic equations (Mulvey et al., 2000), starting with interest rates in several developed countries.

After studying the solutions of the stochastic program for the telecommunication sector, we discovered a particular policy rule (called conditional ratios) that could be implemented in a Monte Carlo simulation with similar results as the stochastic program across the major objectives. A policy that shifted investments to a more conservative allocation when certain triggers occur had a beneficial impact on the overall condition of the sector and the pension plans. The trigger consists of a combination of funding ratio and the ratio of pension assets to market capitalization. To simplify the example, we set the default investment strategy to the 70/30 fixed mix (70% equity and 30% bonds). The conditional strategy keeps assets in the 70/30 growth mode until a potential problem arises as evaluated by the triggers. In fact, over the nine-year horizon, the conditional strategy not only protects the pension surplus but also reduces the NPV of contributions and maximizes the company's value at the end of the nine-year planning horizon (Table 1). In this example, the model develops a sound compromise solution for the diverse stakeholders. However, in general, setting priorities of the multi-objectives for the integrated system presents a complex and potentially controversial issue and provides a direction for future research. Yet, it is important to evaluate the problem in its full capability so that the stakeholders will be able to understand the tradeoffs among the objectives.

Table 1
Means and Ranges of Objective Function Values for the Telecommunication Services Sector Under Two Investment Strategies

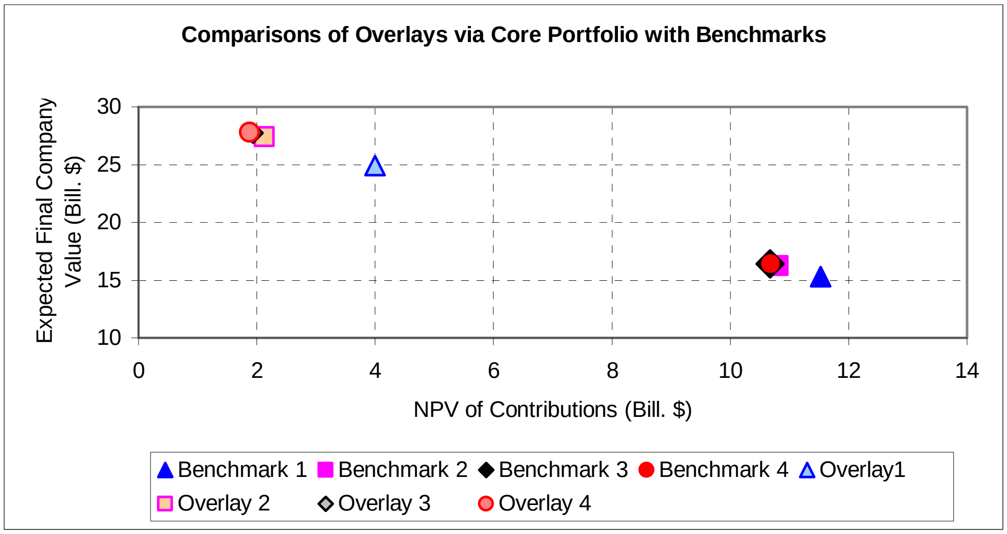
	Expected Final Sector Value			Expected Excess Contribution Penalty Function			Probability of Insolvency		
	Max	Min	Mean	Max	Min	Mean	Max	Min	Mean
Conditional	372.91	335.34	354.23	7.49	0.75	3.48	0.0590	0.0152	0.0381
Benchmark	372.97	335.47	354.38	10.56	1.09	4.93	0.0672	0.0184	0.0433
	Expected NPV of Contributions			**Variability of Contributions**			**Downside Risk on Final Funding ratio**		
	Max	Min	Mean	Max	Min	Mean	Max	Min	Mean
Conditional	31.76	10.27	20.73	18.34	11.19	15.22	0.0800	0.0570	0.0698
Benchmark	31.69	10.23	20.64	19.02	11.38	15.60	0.0823	0.0580	0.0714

The second example case study depicts a more desperate situation than the telecommunication industry. Here, we model a hypothetical automobile company. Not only is the company in a slow growth domain with a large pension obligation (relative to market cap), but the funding ratio is much lower than the telecommunication case, in fact about 70%. We attempt to increase the performance of this company by solving an anticipatory nine-year stochastic program. Unfortunately, the recommended solution does not improve the fortunes of the company and pension system, relative to benchmark strategies such as the 70/30 fixed mix. In many scenarios, the company must make large contributions, weakening the sponsoring company, or it reduces contributions, weakening the pension trust.

Assisting this company requires enhancing investment performance, relative to that obtainable from traditional assets. We employ the concept of an overlay strategy. In this case, the overlay strategies employ the futures markets and trend-following rules, discussed in section II. An overlay strategy can be implemented in several ways. First, we might borrow capital as margin requirements for the futures trades. We call this strategy "overlays with borrowing" (as needed for levering the portfolio). Second, we might employ the core assets as margin requirements for the futures markets. We implemented both strategies in the anticipatory planning model. The calibration of return parameters was carried out as before via maximum likelihood estimates based on historical performance. Rather than conducting a full optimization model, we deploy the overlay strategies as fixed-mix additions to the core policy simulations.

The results appear in Figure 6 and in Table A1 of the Appendix. The first approach (overlays with borrowing) has modest or no impact on overall performance. However, the second strategy improves the long-term objectives for the pension planning problem, such as reducing the net present value and volatility of contribution, while increasing the value of the company at the end of the nine-year horizon. There is a modest increase in volatility over shorter horizons. In summary, the wider diversification and "cheap" leverage available with the second type of overlays improves both the pension trust and the sponsoring company over the long-term. The Appendix provides additional evidence of the advantages of the overlay strategies (Mulvey et al., 2007). Also, overlay strategies based on trend following rules have been successfully implemented by multi-strategy hedge funds, especially the Mt. Lucas Management Company. For the complete discussion regarding this subject, see Mulvey et al. (2007).

Figure 6
Improvements by Employing Overlay Strategies for Illustrative Auto Company[4]



IV. Conclusions and Future Research

The proposed dual strategy provides benefits over a standalone stochastic program or policy simulator. As a significant advantage, a policy simulation model can include many real-world considerations, such as complex regulations, tax laws, and company specific guidelines. These issues are difficult to embed within a continuous stochastic program due to non-differentiability, jump functions, and other complications. On the other hand, a policy simulator requires a predetermined decision/policy rule. Generally, the selection of a policy rule is accomplished by means of long experience and, perhaps, tradition. How can the investor evaluate this rule, relative to other possible rules? The stochastic program (or dynamic program) provides a benchmark. We say that a policy rule that closely approximates the optimal solution values of the stochastic program is "optimal" in so far as it will perform well under the developed restrictive conditions. The dual strategy draws advantages from each of the competing frameworks.

What are promising directions for future research? First, there is much interest in developing a systematic approach for discovering robust policy rules, coming out of the stochastic programming (or stochastic control) solution. We showed that a specific investment rule (conditional ratios) performs quite well, in comparison to alternative

[4] For the detailed explanation of each strategy, see Table A1 in the appendix.

rules and to the stochastic program, for the telecommunication industry. As mentioned, we discovered this rule by carefully studying the solution structure of the 5000 scenario tree. However, there are strong advantages, especially as the size of solvable stochastic program grows, to automating the discovery process. Further research is needed on this topic.

A recurring research topic involves the design of efficient algorithms for solving stochastic program and for optimizing policy simulations. The latter is particularly complex due to the presence of noisy data (with sampling and parameter errors). Remember that an objective function value generated from a policy simulator depicts an output from a statistical experiment and thereby possesses sampling errors. An optimizing algorithm must address sampling errors within the search process. Likewise, policy rules by their nature can give rise to non-convex optimization models. Again, efficient search algorithms are needed.

Last, there has been much recent research involving approximate dynamic programs. Some recent references are: White and Sofge (1992), Bertsekas and Tsitsiklis (1996), Sutton and Barto (1998), Powell (2006) and Infanger (2006). This research may promote further bridges between competing frameworks. The multi-stage financial planning models are among the most difficult in the realm of scientific computing. Therefore, it seems likely that a synthesis of approaches will be needed to find robust solutions to these important societal problems.

References

Arbeleche, S., Dempster, M.A.H., Medova, E.A., Thompson, G.W.P., and Villaverde, M. (2003) 'Portfolio management for pension funds.' *Lecture Notes in Computer Science*, 2690: 462-466

Bertsekas, D. and Tsitsiklis, J. (1996), *Neuro-Dynamic Programming*, Athena Scientific, Belmont, MA.

Boender, G.E.C., van Aalst, P.C. and Heemskerk F. (1998) 'Modelling and management of assets and liabilities of pension plans in the Netherlands', In Ziemba, W.T. and Mulvey, J.M. (Eds.), In: *Worldwide Asset and Liability Modeling* (pp. 561–580). Cambridge University Press, UK

Campbell, J.Y. and Viceira, L.M. (2002) *Strategic Asset Allocation.* Oxford University Press, USA.

Cariño, D.R., Kent, T., Myers, D.H., Stacy, C., Sylvanus, M., Turner, A., Watanabe, K. and Ziemba, W. (1994) 'The Russell-Yasuda Kasai Model: An Asset/Liability Model for a Japanese Insurance Company Using Multistage Stochastic Programming', *Interfaces*, 24, 29–49.

Chan, L.K.C., Jegadeesh, N. and Lakonishok, J. (1996) 'Momentum strategies', *The Journal of Finance*, 51.5, 1681-1713.

Cleary, S. and Inglis, M. (1998) 'Momentum in Canadian stock returns', *Revue Canadienne des Sciences de l'Administraition*, 15.3, 279-291.

Consigli, G. and Dempster, M.A.H. (1998) 'Dynamic Stochastic Programming for Asset-Liability Management', *Annals of Operations Research*, 81, 131–62.

Dantzig, G. and Infanger, G. (1993) 'Multi-Stage Stochastic Linear Programs for Portfolio Optimization', *Annals of Operations Research*, 45, 59–76.

De Bondt, W.F.M. and Thaler, R. (1985) 'Does stock market overreact?' *The Journal of Finance*, 40.3, 793-805.

De Bondt, W.F.M. and Thaler, R. (1987) 'Further evidence on investor overreaction and stock market seasonality', *The Journal of Finance*, 42.3, 557-581.

Demir, I., Muthuswamy, J. and Walter, T. (2004) 'Momentum returns in Australian equities: the influences of size, risk, liquidity and return computation', *Pacific-Basin Finance Journal*, 12, 143-158.

Dempster, M.A.H., Germano, M., Medova, E.A., Rietbergen, M. I., Sandrini, F., Scrowston, M. (2006) Managing guarantees. *Journal of Portfolio Management*, 32, 51-61.

Dert, C.L. (1995) *Asset Liability Management for Pension Funds.* Ph.D. thesis. Erasmus University, Rotterdam, Netherlands.

Fabozzi, F.J., Focardi, S. and Jonas, C. (2004) 'Can Modeling Help Deal with the Pension Funding Crisis?' Working Paper, The Intertek Group.

Fernholz, R. and Shay, B. (1982) 'Stochastic Portfolio Theory and Stock Market Equilibrium', *The Journal of Finance*, 37, 615–24.

Fernholz, R. (2002) *Stochastic Portfolio Theory,* Springer-Verlag, New York, NY

George, T.J. and Hwang, C. (2004) 'The 52-week high and momentum investing', *The Journal of Finance*, 59.5, 2145-2176.

Geyer, A., Herold, W., Kontriner, K. and Ziemba, W.T. (2005) 'The Innovest Austrian pension fund financial planning model InnoALM', working paper, University of British Columbia, Vancouver, BC, Canada.

Hilli, P., Koivu, M., Pennanen T. and Ranne, A. (2003) 'A Stochastic Programming Model for Asset Liability Management of a Finnish Pension Company', *Stochastic Programming E-Print Series*

Infanger, G. (2006) ‘Dynamic asset Allocation Strategies Using a Stochastic Dynamic Programming Approach’, In Zenios, S.A. and Ziemba, W.T. (Eds.), *Handbook of Asset and Liability Management*, Volume 2(1) in *Series of Handbooks in Finance*, Elsevier, Amsterdam.

Jegadeesh, N., Evidence of predictable behavior of security returns, *The Journal of Finance*, Jul 1990, 45.3, 881-898.

Jegadeesh, N. and Titman, S. (1993) ‘Returns to buying winners and selling losers: implications for stock market efficiency’, *The Journal of Finance*, 48.1, 65-91.

Kang, J., Liu, M. and Ni, S.X. (2002) ‘Contrarian and momentum strategies in the China stock market: 1993-2000’, *Pacific-Basin Finance Journal*, 10, 243-265.

Kouwenberg, R. and Zenios, S.A. (2006) ‘Stochastic programming models for asset liability management’, In Zenios, S.A. and Ziemba, W.T. (Eds.), *Handbook of Asset and Liability Management*, Volume 2(1) in *Series of Handbooks in Finance*, Elsevier, Amsterdam.

Lehmann, B.N. (1990) ‘Fads, Martingales, and Market Efficiency’, *Quarterly Journal of Economics*, 105.1, 1-28

Luenberger, D. (1998) *Investment Science.* Oxford University Press, New York, NY.

Merton, R.C. (1969) ‘Lifetime portfolio selection under uncertainty: the continuous-time case’, *Rev. Economics Statistics,* 51, 247-257.

Merton, R.C. (1973) ‘An Intertemporal Capital Asset Pricing Model’, *Econometrica*, 41, 867–887.

Mulvey, J.M., Gould, G. and Morgan, C. (2000) ‘An Asset and Liability Management System for Towers Perrin-Tillinghast’, *Interfaces*, 30, 96–114.

Mulvey, J.M. and Simsek, K.D. (2002) ‘Rebalancing Strategies for Long-term Investors’, In Kontoghiorghes, E. J., Rustem, B. and Siokos, S. (eds), *Computational Methods in Decision Making, Economics and Finance: Optimization Models*, Kluwer, Netherlands.

Mulvey, J.M., Pauling, B. and Madey, R.E. (2003a) ‘Advantages of Multiperiod Portfolio Models’, *Journal of Portfolio Management*, 29, 35–45.

Mulvey, J.M., Simsek, K.D., Zhang, Z., Fabozzi, F.J. and Pauling, B. (2008) ‘Assisting Defined-Benefit Pension Plans’, *Operations Research*, to appear

Mulvey, J.M., Simsek, K.D., Zhang, Z. and Holmer, M. (2004a) ‘Preliminary Analysis of Defined-Benefit Pension Plans’, Princeton University Report, Department of OR and Financial Engineering.

Mulvey, J.M., Kaul, S.S.N. and Simsek, K.D. (2004b) ‘Evaluating a trend-following commodity index for multi-period asset allocation’, *J. Alternative Investments*, 54-69.

Mulvey, J.M. (2005) ‘Essential portfolio theory’, *A Rydex Investment White Paper,* Princeton University

Mulvey, J.M., Kim, W.C. (2007a), ‘Constructing a Portfolio of Industry-Level Momentum Strategies across Global Equity Markets’, Princeton University Report, Department of OR and Financial Engineering.

Mulvey, J.M., Kim, W.C. (2007b), ‘The Role of Alternative Assets for Optimal Portfolio Construction’, *Wiley Encyclopedia on Risk Assessment*, to appear

Mulvey J.M., Ural, C. and Zhang, Z. (2007) ‘Improving Performance for Long-Term Investors: Wide Diversification, Leverage and Overlay Strategies’, *Quantitative Finance*, 7.2:1-13

Peskin, M.W. (1997) ‘Asset Allocation and Funding Policy for Corporate-Sponsored Defined-Benefit Pension Plans’, *Journal of Portfolio Management*, 23, 66–73.

Pflug, G.Ch., Dockner, E., Swietanowski, A. and Moritsch H. (2000) ‘The AURORA financial management system: from model design to parallel implementation’, *Annals of Operations Research*, 99, 189 – 206.

Pflug, G.Ch. and Swietanowski, A. (2000) ‘Asset-liability optimization for Pension Fund Management’, *Operations Research Proceedings 2000,* Springer-Verlag, 124 – 135

Rouwenhorst, K.G. (1998) ‘International momentum strategies’, *The Journal of Finance,* 53.1, 267-284.

Rudolf, M. and Ziemba, W.T. (2004) ‘Intertemporal Asset-Liability Management’, *Journal of Economic Dynamics and Control* 28:4, 975-990.

Samuelson, P.A. (1969) ‘Lifetime portfolio selection by dynamic stochastic programming’, *Rev. Economics Statistics,* 51, 239-246.

Sutton, R. and Barto, A. (1998), *Reinforcement Learning*, The MIT Press, Cambridge, Massachusetts.

Powell, W. (2006), *Approximate Dynamic Programming: Solving the curses of dimensionality*, Princeton University

White, D.A. and Sofge, D.A. (1992), *Handbook of Intelligent Control*, Von Nostrand Reinhold, New York, NY.

Zenios, S.A. and Ziemba, W.T. (2006) *Handbook of Asset and Liability Management*, Volume 2(1) in *Series of Handbooks in Finance*, Elsevier, Amsterdam.

Zenios, S.A. and Ziemba, W.T. (2007) *Handbook of Asset and Liability Management*, Volume 2(2) in *Series of Handbooks in Finance*, Elsevier, Amsterdam.

Zhang, Z. (2006) *Stochastic Optimization for Enterprise Risk Management*. Doctoral Dissertation. Department of Operations Research and Financial Engineering, Princeton University, Princeton, NJ.

Ziemba, W.T. and Mulvey, J.M. (1998) *Worldwide Asset and Liability Modeling*. Cambridge University Press, UK.

Ziemba, W.T. (2003) *The Stochastic Programming Approach to Asset-Liability and Wealth Management*. AIMR-Blackwell.

Appendix

Figure A1
Performance of Dynamic Diversification Portfolio

Dynamic diversification portfolio is an equally weighted fixed mix portfolio of 30 momentum strategies -- five regions, six settings. Each number next to a point on the line represents leverage. 3-month U.S. T-bill is used. The sample period is 1980~2006.

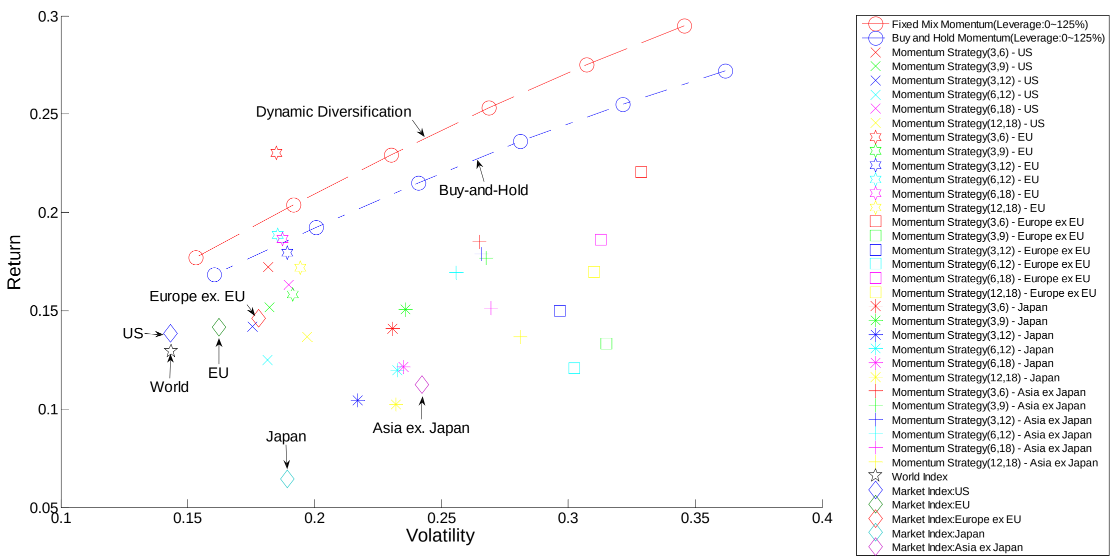


Table A1
Projections of Illustrative Auto Company under Various Benchmarks and under two Overlay Strategies

This table displays a list of summary projection results of the illustrative auto company (section 3) under three set of strategies: 1) four fixed-mix strategies as benchmarks with varying degrees of risk aversion; 2) four overlay strategies via the core portfolio (70% in S&P 500 index and 30% in STRIPS); and 3) four overlay strategies via borrowing at the risk-free treasury bill rates. The overlay strategy via the core portfolio outperforms the others on most dimensions.

	Benchmark 1	**Benchmark 2**	**Benchmark 3**	**Benchmark 4**
	S&P 500 0.4	**S&P 500** 0.6	**S&P 500** 0.7	**S&P 500** 0.8
	25-Yr STRIPS 0.6	**25-Yr STRIPS** 0.4	**25-Yr STRIPS** 0.3	**25-Yr STRIPS** 0.2
	Commodity 0	**Commodity** 0	**Commodity** 0	**Commodity** 0
	Currency 0	**Currency** 0	**Currency** 0	**Currency** 0
	Fixed Income 0	**Fixed Income** 0	**Fixed Income** 0	**Fixed Income** 0
Expected Final Company Value (Bill. $)	15.331	16.227	16.425	16.435
Expected Final Plan Surplus (Bill. $)	-8.280	-8.188	-7.749	-7.027
Expected Final Funding Ratio	92.15%	92.38%	92.87%	93.64%
Semi-Standard Deviation of Final Funding Ratio	6.80%	6.75%	7.50%	8.68%
Standard Deviation of Final Funding Ratio	10.39%	10.01%	11.31%	13.43%
Standard Deviation of Funding Ratio across All Periods	12.86%	12.75%	13.45%	14.67%
NPV of Contributions (Bill. $)	11.528	10.810	10.668	10.678
Volatility of Contributions (Bill. $)	3.609	3.533	3.610	3.754
Probability of Any Excess Contribution	49.16%	49.68%	50.36%	51.22%
Excess Contribution Penalty Function (Bill. $)	6.332	6.212	6.418	6.801
Probability of Insolvency	45.74%	45.88%	46.32%	46.64%
	Overlay1	**Overlay 2**	**Overlay 3**	**Overlay 4**
	S&P 500 0.7	**S&P 500** 0.7	**S&P 500** 0.7	**S&P 500** 0.7
	25-Yr STRIPS 0.3	**25-Yr STRIPS** 0.3	**25-Yr STRIPS** 0.3	**25-Yr STRIPS** 0.3
	Commodity 0.5	**Commodity** 0.75	**Commodity** 1	**Commodity** 1
	Currency 0.5	**Currency** 1.25	**Currency** 2	**Currency** 1
	Fixed Income 0.5	**Fixed Income** 1.75	**Fixed Income** 1	**Fixed Income** 1
	Overlays via Core Portfolio			
Expected Final Company Value (Bill. $)	24.930	27.419	27.746	27.791
Expected Final Plan Surplus (Bill. $)	1.877	30.287	45.316	28.193
Expected Final Funding Ratio	101.42%	123.02%	133.48%	121.86%
Semi-Standard Deviation of Final Funding Ratio	12.76%	33.22%	45.69%	31.53%
Standard Deviation of Final Funding Ratio	22.07%	58.37%	82.21%	55.56%
Standard Deviation of Funding Ratio across All Periods	18.60%	40.74%	55.04%	38.79%
NPV of Contributions (Bill. $)	3.992	2.126	1.932	1.880
Volatility of Contributions (Bill. $)	2.034	1.568	1.523	1.416
Probability of Any Excess Contribution	25.86%	13.64%	11.46%	12.44%
Excess Contribution Penalty Function (Bill. $)	3.484	2.800	2.770	2.524
Probability of Insolvency	36.46%	33.40%	32.98%	33.14%
	Overlays via Borrowing			
Expected Final Company Value (Bill. $)	17.977	14.052	15.158	17.773
Expected Final Plan Surplus (Bill. $)	-6.210	-3.688	0.343	-1.330
Expected Final Funding Ratio	94.14%	96.07%	99.73%	98.37%
Semi-Standard Deviation of Final Funding Ratio	9.87%	17.13%	21.10%	16.69%
Standard Deviation of Final Funding Ratio	15.76%	29.40%	39.90%	30.43%
Standard Deviation of Funding Ratio across All Periods	16.09%	24.85%	30.46%	24.52%
NPV of Contributions (Bill. $)	9.340	12.330	11.580	9.503
Volatility of Contributions (Bill. $)	3.506	4.807	4.723	3.937
Probability of Any Excess Contribution	44.24%	50.22%	45.94%	41.36%
Excess Contribution Penalty Function (Bill. $)	6.326	9.697	9.602	7.564
Probability of Insolvency	44.40%	49.20%	47.76%	44.64%